# Year 12 Methods Unit 3 Revision Lecture Monash University presented by Kevin McMenamin

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# 1 Graphs

### 16 types of graph—put in reference book:

1. truncus	9. semicircle
2. hyperbola	10. tan
3. sqrt	11. sin
4. parabola	12. cos
5. cubic	13. log
6. quartic	14. exp
7. linear	15. $x^{\frac{a}{b}}$
8. circle	16. $x^{\frac{-a}{b}}$

### 1.1 Power functions

• In first quadrant, shape of graph for  $x > 0 \cap y > 0$  is either  $\sqrt{x}$  or  $x^2$ 

# 1.2 Features of graphs

- Asymptotes
- Intercepts
- Stationary points
- Endpoints
- Other critical points
- Continuous or discontinuous

## Key points

- All transformations can be described by matrices
- Inverse is a transformation
- Memorise approximate values of  $e, \pi, \sqrt{2}, \sqrt{3}$
- Put 16 base graphs in reference book

# 2 Transformations

 $\mathrm{Order:}\qquad \mathbf{Reflect}\longrightarrow \mathbf{Dilate}\longrightarrow \mathbf{Translate}$ 

# 2.1 Two forms

- note a and b can be positive or negative
- check validity of solutions for logarithms
- results in transformed equation y' = f'(x)

$$y' = a \cdot f(\frac{1}{b}(x'-c)) + d$$
$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} b & 0\\0 & a \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix} + \begin{bmatrix} c\\d \end{bmatrix}$$

#### Key points

- All transformations can be described by matrices
- Inverse is a transformation
- Check validity of  $\log_a x$  solutions/transformations

# 3 Calculus

Possible questions:

- Average rate of change
- Instantaneous rate of change
- Tangent line
- Normal line
- Features of gradient function
  - Degree
  - Orientation
  - Format
  - Turning points
  - Inflection points
  - Asymptotes
- Find original function from derivative
   → Use information to find unknowns
- Application questions e.g. Pythagoras, trig. functions, measurement, given eqn

### 3.1 Integration

### 3.1.1 Polynomials

$$f(x) = \int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c \,, \quad n \neq -1$$
$$f(x) = \int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c \,, \quad n \neq -1$$

#### 3.1.2 Exponentials

$$f(x) = \int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$$

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#### 3.1.3 Logarithms

ignore modulus for methods

$$f(x) = \int \frac{1}{x} dx = \ln |x| + c$$

$$f(x) = \int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + c$$

$$f(x) - \int \frac{h'(x)}{h(x)} dx = \ln |h(x)| + c$$
(general form)

#### 3.1.4 Trigonometric functions

$$f(x) = \int \cos(ax+b) \, dx = \frac{1}{a} \sin(ax+b) + c$$
$$f(x) = \int \sin(ax+b) \, dx = -\frac{1}{a} \sin(ax+b) + c$$
$$f(x) = \int \sec^2(ax+b) \, dx = \frac{1}{a} \tan(ax+b) + c$$

## 3.2 Area under curves

- Upper rectangles (overestimate) vs. lower rectangles (underestimate)
- Rotate (invert) graph to make it easier, e.g.  $y = \sqrt{x} \longrightarrow x = y^2$

### Key points

- For an antiderivative,  $+c \quad \forall c \in \mathbb{R}$  is also acceptable
- Practice multi-part problems e.g:
  - a) Let  $f : \mathbb{R} \to \mathbb{R}$ ,  $f(x) = x \sin x$ . Find f'(x).
  - b) Use the result of (a) to find the value of  $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \cos x \, dx$  in the form  $a\pi + b$ .

# 4 Probability

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$\Pr(A \cup B) = 0$$
(mute

mutually exclusive)

## 4.1 Conditional probability

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$Pr(A \cap B) = Pr(A|B) \times Pr(B)$$

$$Pr(A \cap B) = Pr(A) \times Pr(B)$$
(independent events)

### 4.2 Discrete random distributions

Any experiment or activity involving chance will have a probability associated with each result or *outcome*. If the outcomes have a reference to **discrete numeric values** (outcomes that can be counted), and the result is unknown, then the activity is a *discrete random probability distribution*.

### 4.2.1 Discrete probability distributions

If an activity has outcomes whose probability values are all positive and less than one ( $\implies 0 \le p(x) \le 1$ ), and for which the sum of all outcome probabilities is unity ( $\implies \sum p(x) = 1$ ), then it is called a *probability distribution* or *probability mass* function.

- Probability distribution graph a series of points on a cartesian axis representing results of outcomes. Pr(X = x) is on y-axis, x is on x axis.
- Mean  $\mu$  measure of central tendency. *Balance point* or *expected value* of a distribution. Centre of a symmetrical distribution.
- Variance  $\sigma^2$  measure of spread of data around the mean. Not the same magnitude as the original data. Represented by  $\sigma^2 = \operatorname{Var}(x) = \sum (x = \mu)^2 \times p(x) = \sum (x \mu)^2 \times \Pr(X = x)$ . Alternatively:  $\sigma^2 = \operatorname{Var}(X) = \sum x^2 \times p(x) \mu^2$
- Standard deviation  $\sigma$  measure of spread in the original magnitude of the data. Found by taking square root of the variance:  $\sigma = \operatorname{sd}(X) = \sqrt{\operatorname{Var}(X)}$

# 4.3 Binomial distribution (Bernoulli trials)

A type of discrete probability distribution. This distribution has the following characteristics:

- 1. Samples are taken from a population size that remains constant (sampling with replacement)
- 2. Every result or trial can be classed as either a success or failure
- 3. The probability of a success is the same from one trial to the next, notated by p
- 4. The probability of a failure is the complement of the probability of a success, notated by 1 p
- 5. There are a finite number of trials that define the sample size, notated by n

#### 4.3.1 Bernoulli trials

Same properties as above. Number of successes in a finite number of Bernoulli trials is defined as the **binomial distribution**. The distribution can take the form:

$$X \sim \operatorname{Bi}(n, p)$$

Then, the probability values for each value of X follow the rule:

$$p(x) = \begin{bmatrix} n \\ x \end{bmatrix} (p)^x (1-p)^{n-x}$$

### 4.4 Continuous random distributions

If the outcomes of an activity have a reference to *continuous numeric* values (outcomes that can be measured), then the activity is associated with a **continuous probability distribution**. The probabilities are calculated by finding the area under the graph between the required x values (integrate).

The probability of a single *outcome value* does not exist for continuous probability distributions.

#### 4.5 Continuous probability distributions

If an experiment or activity has a **function** whose values are all positive ( $\implies f(x) \ge 0 \forall x$ ), and for which the area under the graph between the lowest outcome value and the greatest outcome value is unity ( $\implies \int_{\text{lower}}^{\text{upper}} f(x) \, dx = 1$ ), then it is called a **probability density function**.

Example probability density function:  $f(x) = \begin{cases} k(9-x^2), & 0 \le x \le 3\\ 0, & \text{elsewhere} \end{cases}$ 

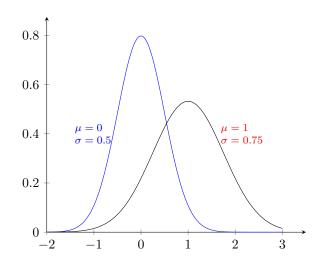


Figure 1: Two general normal distributions

#### 4.6 Normal distributions

A very specific and special continuous probability distribution. Characteristics:

- Many sets of data occurring naturally and taken randomly will have a normal distribution
- No single outcome value can be calculated
- Probabilities are found between certain outcome values of the distribution
- The values of the distribution are symmetrical around the mean  $(\mu)$  and form a bell-shaped curve
- The distribution is best described using its central or mean value,  $\mu$ , and its measure of spread,  $\sigma$
- The distribution can take the form  $X \sim N(\mu, \sigma^2)$

