

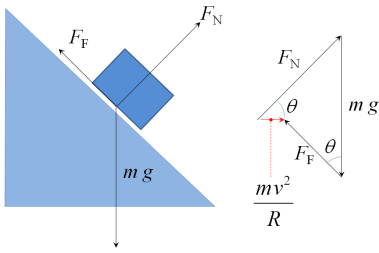
# 1 Motion

$m/s \times 3.6 = km/h$

## Inclined planes

$F = mg \sin \theta - F_{frict} = ma$

## Banked tracks



$\theta = \tan^{-1} \frac{v^2}{rg}$

$\Sigma F$  always acts towards centre (horizontally)

$\Sigma F = F_{norm} + F_g = \frac{mv^2}{r} = mg \tan \theta$

Design speed  $v = \sqrt{gr \tan \theta}$

$n \sin \theta = mv^2 \div r, \quad n \cos \theta = mg$

## Work and energy

$W = Fs = F s \cos \theta = \Delta \Sigma E$

$E_K = \frac{1}{2}mv^2$  (kinetic)

$E_G = mgh$  (potential)

$\Sigma E = \frac{1}{2}mv^2 + mgh$  (energy transfer)

## Horizontal circular motion

$v = \frac{2\pi r}{T}$

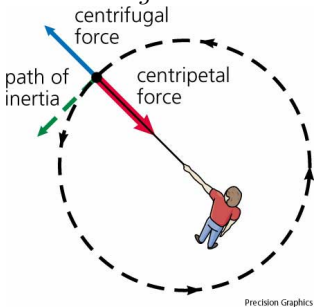
$f = \frac{1}{T}, \quad T = \frac{1}{f}$

$a_{centrip} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$\Sigma F, a$  towards centre,  $v$  tangential

$\Sigma F = F_{centrip} = \frac{mv^2}{r} = \frac{4\pi^2 r m}{T^2} =$

$T \sin \theta = mg \tan \theta$



## Vertical circular motion

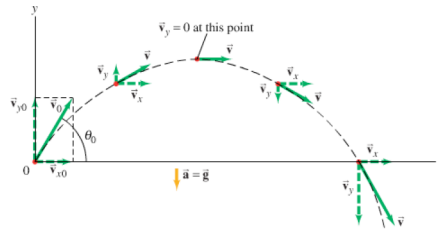
$T + mg = \frac{mv^2}{r}, v = \sqrt{rg}$  (top)

$T - mg = \frac{mv^2}{r}, v = \sqrt{2rg}$  (bottom)

$E_{K\text{bottom}} = E_{K\text{top}} + mgh$

## Projectile motion

- $v_x$  is constant:  $v_x = \frac{s}{t}$
  - use suvat to find  $t$  from  $y$ -component
  - vertical component gravity:  $a_y = -g$
- vectors
- max height  $\Delta x$  at  $t$
- height at  $t$
- time of flight
- horiz. range
- $v = \sqrt{v_x^2 + v_y^2}$
- $h = \frac{u^2 \sin^2 \theta}{2g}$
- $x = ut \cos \theta$
- $y = ut \sin \theta - \frac{1}{2}gt^2$
- $t = \frac{2u \sin \theta}{g}$
- $d = \frac{v^2}{g} \sin \theta$



## Pulley-mass system

$a = \frac{m_2 g}{m_1 + m_2}$  where  $m_2$  is suspended

$\Sigma F = m_2 g - m_1 g = \Sigma ma$  (solve)

## Graphs

- Force-time:  $A = \Delta \rho$
- Force-disp:  $A = W$
- Force-ext:  $m = k, \quad A = E_{spr}$
- $F_g$ -dist:  $A = \Delta gpe$
- Field-dist:  $A = \Delta gpe / kg$

## Hooke's law

$F = -kx$  (intercepts origin)

elastic potential energy  $= \frac{1}{2}kx^2$

$x = \frac{2mg}{k}$

Vertical:  $\Delta E = \frac{1}{2}kx^2 + mgh$

## Motion equations

no

$v = u + at$   $x$

$x = \frac{1}{2}(v + u)t$   $a$

$x = ut + \frac{1}{2}at^2$   $v$

$x = vt - \frac{1}{2}at^2$   $u$

$v^2 = u^2 + 2ax$   $t$

## Momentum

$\rho = mv$

impulse  $= \Delta \rho, \quad F \Delta t = m \Delta v$

$\Sigma(mv_0) = (\Sigma m)v_1$  (conservation)

if elastic:

$\sum_{i=1}^n E_K(i) = \sum_{i=1}^n (\frac{1}{2}m_i v_{i0}^2) = \frac{1}{2} \sum_{i=1}^n (m_i) v_f^2$

# 2 Relativity

## Postulates

- Laws of physics are constant in all inertial reference frames
- Speed of light  $c$  is the same to all observers (Michelson-Morley)

$\therefore t$  must dilate as speed changes

**high-altitude particles:**  $t$  dilation means more particles reach Earth than expected (half-life greater when obs. from Earth)

**Inertial reference frame**  $a = 0$

**Proper time**  $t_0$  | **length**  $l_0$  measured by observer in same frame as events

## Lorentz factor

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad v = c \sqrt{1 - \frac{1}{\gamma^2}}$

$t = t_0 \gamma$  ( $t$  longer in moving frame)

$l = \frac{l_0}{\gamma}$  ( $l$  contracts  $\parallel v$ : shorter in moving frame)

$m = m_0 \gamma$  (mass dilation)

## Energy and work

$E_{rest} = mc^2, \quad E_K = (\gamma - 1)mc^2$

$E_{total} = E_K + E_{rest} = \gamma mc^2$

$W = \Delta E = \Delta mc^2 = (\gamma - 1)m_{rest}c^2$

### Relativistic momentum

$$\rho = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mv = \gamma \rho_0$$

$\rho \rightarrow \infty$  as  $v \rightarrow c$

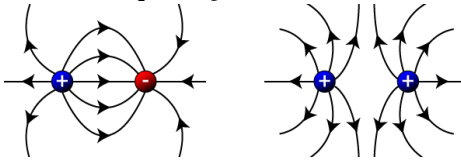
$v = c$  is impossible (requires  $E = \infty$ )

$$v = \frac{\rho}{m\sqrt{1 + \frac{\rho^2}{m^2 c^2}}}$$

### 3 Fields and power

#### Non-contact forces

- electric (dipoles & monopoles)
- magnetic (dipoles only)
- gravitational (monopoles only,  $F_g = 0$  at mid, attractive only)
- monopoles: lines towards centre
- dipoles: field lines  $+ \rightarrow -$  or  $N \rightarrow S$  (two magnets) or  $\rightarrow N$  (single)
- closer field lines means larger force
- dot: out of page, cross: into page
- +ve corresponds to N pole
- Inv. sq.  $\frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2$



#### Gravity

$$F_g = G \frac{m_1 m_2}{r^2} \quad (\text{grav. force})$$

$$g = \frac{F_g}{m_2} = G \frac{m_1}{r^2} \quad (\text{field of } m_1)$$

$$E_g = mg\Delta h \quad (\text{gpe})$$

$$W = \Delta E_g = Fx \quad (\text{work})$$

$$w = m(g - a) \quad (\text{app. weight})$$

#### Satellites

$$v = \sqrt{\frac{GM}{r}} = \sqrt{gr} = \frac{2\pi r}{T}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} \quad (\text{period})$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \quad (\text{radius})$$

### Magnetic fields

- field strength  $B$  measured in tesla
- magnetic flux  $\Phi$  measured in weber
- charge  $q$  measured in coulombs
- emf  $\mathcal{E}$  measured in volts

$$F = qvB \quad (F \text{ on moving } q)$$

$$F = IlB \quad (F \text{ of } B \text{ on } I)$$

$$B = \frac{mv}{qr} \quad (\text{field strength on } e^-)$$

$$r = \frac{mv}{qB} \quad (\text{radius of } q \text{ in } B)$$

if  $B \perp A, \Phi \rightarrow 0$  , if  $B \parallel A, \Phi = 0$

### Electric fields

$$F = qE (= ma) \quad (\text{strength})$$

$$F = k \frac{q_1 q_2}{r^2} \quad (\text{force between } q_{1,2})$$

$$E = k \frac{q}{r^2} \quad (\text{field on point charge})$$

$$E = \frac{V}{d} \quad (\text{field between plates})$$

$$F = BIl \quad (\text{force on a coil})$$

$$\Phi = B_{\perp} A \quad (\text{magnetic flux})$$

$$\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} = Blv \quad (\text{induced emf})$$

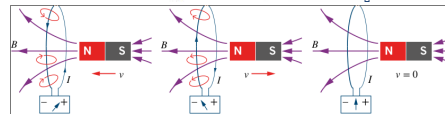
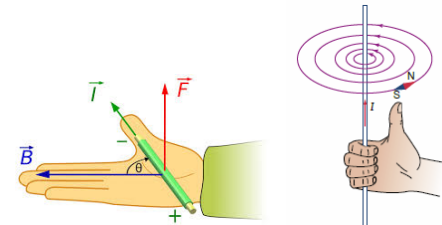
$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \quad (\text{xfmr coil ratios})$$

**Lenz's law:**  $I_{\text{emf}}$  opposes  $\Delta \Phi$

(emf creates  $I$  with associated field that opposes  $\Delta \Phi$ )

**Eddy currents:** counter movement within a field

**Right hand grip:** thumb points to  $I$  (single wire) or  $N$  (solenoid / coil)



**Flux-time graphs:**  $m \times n = \text{emf}$ . If  $f$  increases, ampl. &  $f$  of  $\mathcal{E}$  increase

**Xfmr** core strengthens & focuses  $\Phi$

### Particle acceleration

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$e^-$  accelerated with  $x \text{ V}$  is given  $x \text{ eV}$

$$W = \frac{1}{2}mv^2 = qV \quad (\text{field or points})$$

$$v = \sqrt{\frac{2qV}{m}} \quad (\text{velocity of particle})$$

Circular path:  $F \perp B \perp v$

### Power transmission

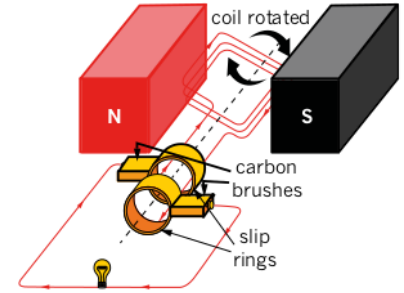
$$V_{\text{rms}} = \frac{V_p}{\sqrt{2}} = \frac{V_{p \rightarrow p}}{2\sqrt{2}}$$

$$P_{\text{loss}} = \Delta VI = I^2 R = \frac{\Delta V^2}{R}$$

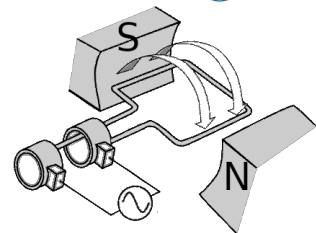
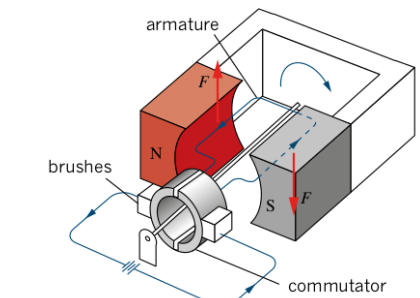
$$V_{\text{loss}} = IR$$

Use high- $V$  side for correct  $|V_{\text{drop}}|$

- Parallel  $V$  is constant
- Series  $V$  shared within branch



### Motors



Force on I-carrying wire, not Cu

$F = 0$  for front back of coil (parallel)

Any angle  $> 0$  will produce force

**DC:** split ring (two halves)

**AC:** slip ring (separate rings with constant contact)

## 4 Waves

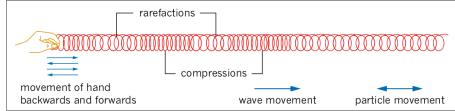
**nodes:** fixed on graph

**amplitude:** max disp. from  $y = 0$

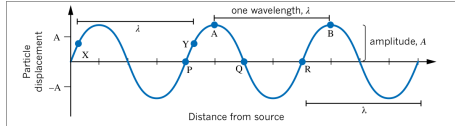
**rarefactions and compressions**

**mechanical:** transfer of energy without net transfer of matter

### Longitudinal (motion || wave)



### Transverse (motion ⊥ wave)



$$T = \frac{1}{f} \quad (\text{period: time for one cycle})$$

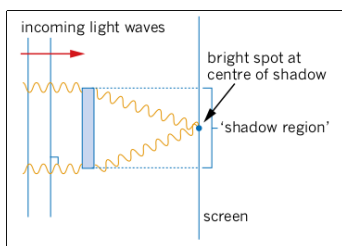
$$v = f\lambda \quad (\text{speed: displacement / sec})$$

$$f = \frac{c}{\lambda} \quad (\text{for } v = c)$$

## Doppler effect

When  $P_1$  approaches  $P_2$ , each wave  $w_n$  has slightly less distance to travel than  $w_{n-1}$ .  $w_n$  reaches observer sooner than  $w_{n-1}$  ("apparent"  $\lambda$ ).

## Interference



Poisson's spot supports wave theory (circular diffraction)

**Standing waves** - constructive int. at resonant freq. Rebound from ends.

**Coherent** - identical frequency, phase, direction (ie strong directional). e.g. laser

**Incoherent** - e.g. incandescent/LED

## Harmonics

1st harmonic = fundamental

**for nodes at both ends:**

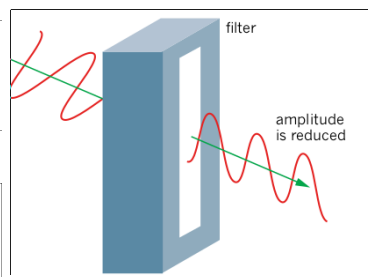
$$\lambda = 2l \div n \quad f = nv \div 2l$$

**for node at one end ( $n$  is odd):**

$$\lambda = 4l \div n \quad f = nv \div 4l$$

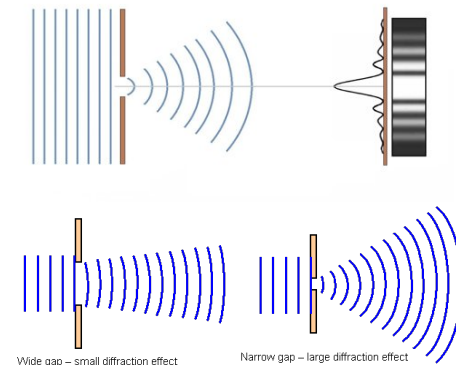
alternatively,  $\lambda = \frac{4l}{2n-1}$  where  $n \in \mathbb{Z}$  and  $n+1$  is the next possible harmonic

## Polarisation



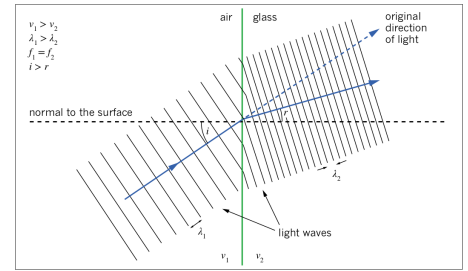
Transverse only. Reduces total  $A$ .

## Diffraction



- Constructive:  $pd = n\lambda, n \in \mathbb{Z}$
- Destructive:  $pd = (n - \frac{1}{2})\lambda, n \in \mathbb{Z}$
- Path difference:  $\Delta x = \frac{\lambda l}{d}$  where  $l$  = distance from source to observer,  $d$  = separation between each wave source (e.g. slit) =  $S_1 - S_2$
- diffraction  $\propto \frac{\lambda}{d} \propto$  fringe spacing
- $d(|\vec{S}_1 \vec{W}| - |\vec{S}_2 \vec{W}|) = d\Delta x = \lambda l$
- significant diffraction when  $\frac{\lambda}{\Delta x} \geq 1$
- diffraction creates distortion (electron optical microscopes)

## Refraction



When a medium changes character, light is *reflected*, *absorbed*, and *transmitted*.  $\lambda$  changes, not  $f$ .

angle of incidence  $\theta_i$  = angle of reflection  $\theta_r$

$$\text{Critical angle } \theta_c = \sin^{-1} \frac{n_2}{n_1}$$

Snell's law  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$$v_1 \div v_2 = \sin \theta_1 \div \sin \theta_2$$

$$n_1 v_1 = n_2 v_2$$

$$n = \frac{c}{v}$$

## 5 Light and Matter

### Planck's equation

$$E = hf = \frac{hc}{\lambda} = \rho c = qV$$

$$h = 6.63 \times 10^{-34} \text{ Js} = 4.14 \times 10^{-15} \text{ eVs}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

### De Broglie's theory

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2W}{m}}}$$

$$\rho = \frac{hf}{c} = \frac{h}{\lambda} = mv, \quad E = \rho c$$

$$v = \sqrt{2E_K \div m}$$

- cannot confirm with double-slit (slit  $< r_{\text{proton}}$ )
- confirmed by e- and x-ray patterns

### Force of electrons

$$F = \frac{2P_{\text{in}}}{c}$$

$$\text{photons / sec} = \frac{\text{total energy}}{\text{energy / photon}}$$

$$= \frac{P_{\text{in}}\lambda}{hc} = \frac{P_{\text{in}}}{hf}$$

## X-ray electron interaction

- e- stable if  $mvr = n \frac{h}{2\pi}$  where  $n \in \mathbb{Z}$  and  $r$  is radius of orbit
- $\therefore 2\pi r = n \frac{h}{mv} = n\lambda$  (circumference)
- if  $2\pi r \neq n \frac{h}{mv}$ , no standing wave
- if e- = x-ray diff patterns,  $E_{e-} = \frac{p^2}{2m} = (\frac{h}{\lambda})^2 \div 2m$

## Photoelectric effect

- $V_{\text{supply}}$  does not affect photocurrent
- $V_{\text{sup}} > 0$ : attracted to +ve
- $V_{\text{sup}} < 0$ : attracted to -ve,  $I \rightarrow 0$
- $v$  of e- depends on shell
- max  $I$  (not  $V$ ) depends on intensity

## Threshold frequency $f_0$

min  $f$  for photoelectron release. if  $f < f_0$ , no photoelectrons.

## Work function $\phi = hf_0$

min  $E$  for photoelectron release. determined by strength of bonding. Units: eV or J.

## Kinetic energy $E_K = hf - \phi = qV_0$

$V_0 = E_K$  in eV

dashed line below  $E_K = 0$

## Stopping potential $V_0$ for min $I$

$$V_0 = h_{\text{eV}}(f - f_0)$$

Opposes induced photocurrent

## Graph features

	$m$	$x$ -int	$y$ -int
$f \cdot E_K$	$h$	$f_0$	$-\phi$
$V \cdot I$		$V_0$	intensity
$f \cdot V$	$\frac{h}{q}$	$f_0$	$\frac{-\phi}{q}$

## Spectral analysis

- $\Delta E = hf = \frac{hc}{\lambda}$  between ground / excited state
- $E$  and  $f$  of photon:  $E_2 - E_1 = hf = \frac{hc}{\lambda}$
- Ionisation energy - min  $E$  required to remove e-
- EMR is absorbed/emitted when  $E_{K\text{-in}} = \Delta E_{\text{shells}}$  (i.e.  $\lambda = \frac{hc}{\Delta E_{\text{shells}}}$ )
- No. of lines - include all possible states

## Uncertainty principle

$$\Delta x \approx \frac{\text{slit width}}{2}$$

measurement:  $\rho$  transferred to e- slit: possibility of diff. before slit

## Wave-particle duality

### wave model

- cannot explain photoelectric effect
- $f$  is irrelevant to photocurrent
- predicts delay between incidence and ejection
- speed depends on medium
- supported by bright spot in centre
- $\lambda = \frac{hc}{E}$

### particle model

- explains photoelectric effect
- rate of photoelectron release  $\propto$  intensity
- no time delay - one photon releases one electron
- double slit: photons interact. interference pattern still appears when a dim light source is used so that only one photon can pass at a time
- light exerts force

- light bent by gravity
- quantised energy
- $\lambda = \frac{h}{p}$

## 6 Experimental design

### Absolute uncertainty $\Delta$

(same units as quantity)

$$\Delta(m) = \frac{\mathcal{E}(m)}{100} \cdot m$$

$$(A \pm \Delta A) + (B \pm \Delta A) = (A+B) \pm (\Delta A + \Delta B)$$

$$(A \pm \Delta A) - (B \pm \Delta A) = (A-B) \pm (\Delta A + \Delta B)$$

$$c(A \pm \Delta A) = cA \pm c\Delta A$$

### Relative uncertainty $\mathcal{E}$ (unitless)

$$\mathcal{E}(m) = \frac{\Delta(m)}{m} \cdot 100$$

$$(A \pm \mathcal{E}A) \cdot (B \pm \mathcal{E}B) = (A \cdot B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A) \div (B \pm \mathcal{E}B) = (A \div B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A)^n = (A^n \pm n\mathcal{E}A)$$

$$c(A \pm \mathcal{E}A) = cA \pm \mathcal{E}A$$

Uncertainty of a measurement is  $\frac{1}{2}$  the smallest division

**Precision** - concordance of values

**Accuracy** - closeness to actual value

**Random errors** - unpredictable, reduced by more tests

**Systematic errors** - not reduced by more tests

**Uncertainty** - margin of potential error

**Error** - actual difference

**Hypothesis** - can be tested experimentally

**Model** - evidence-based but indirect representation

