Polynomials

Factorising

Quadratics

Quadratics: $x^2 + bx + c = (x+m)(x+n)$ where $mn = c$, $m + n = b$
Difference of squares: $a^2 - b^2 = (a-b)(a+b)$
Perfect squares: $a^2 \pm 2ab + b^2 = (a \pm b^2)$
Completing the square (monic): $x^2 + bx + c = (x + \frac{b}{2})^2 + c - \frac{b^2}{4}$
Completing the square (non-monic): $ax^2 + bx + c = a(x - \frac{b}{2a})^2 + c - \frac{b^2}{4a}$
Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $\Delta = b^2 - 4ac$ (if Δ is a perfect square, rational roots)

Cubics

Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Perfect cubes: $a^3 \pm 3a^2b + 3ab^2 \pm b^3 = (a \pm b)^3$

Linear and quadratic graphs

Forms of linear equations

$y = mx + c$ where m is gradient and c is y-intercept
$\frac{x}{a} + \frac{y}{b} = 1$ where m is gradient and (x_1, y_1) lies on the graph
$y - y_1 = m(x - x_1)$ where $(a, 0)$ and $(0, b)$ are x- and y-intercepts

Line properties

Parallel lines: $m_1 = m_2$
Perpendicular lines: $m_1 \times m_2 = -1$
Distance: $\vec{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Cubic graphs

$$y = a(x-b)^3 + c$$

- $m = 0$ at *stationary point of inflection*
- in form $y = (x-a)^2(x-b)$, local max at $x = a$, local min at $x = b$
- in form $y = a(x-b)(x-c)(x-d)$: x-intercepts at b, c, d

Quartic graphs

Forms of quadratic equations

$y = ax^4$
$y = a(x-b)(x-c)(x-d)(x-e)$
$y = ax^4 + cd^2 (c \geq 0)$
$y = ax^2(x-b)(x-c)$
$y = a(x-b)^2(x-c)^2$
$y = a(x-b)(x-c)^3$

Literal equations

Equations with multiple pronumerals. Solutions are expressed in terms of pronumerals (parameters)

Simultaneous equations (linear)

- **Unique solution** - lines intersect at point
- **Infinitely many solutions** - lines are equal
- **No solution** - lines are parallel

Solving $\begin{cases} px + qy = a \\ rx + sy = b \end{cases}$ **for one, infinite and no solutions**

where all coefficients are known except for one, and a, b are known

1. Write as matrices: $\begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$
2. Find determinant of first matrix: $\Delta = ps - qr$
3. Let $\Delta = 0$ for number of solutions $\neq 1$
   or let $\Delta \neq 0$ for one unique solution.
4. Solve determinant equation to find variable
   - *— for infinite/no solutions: —*
5. Substitute variable into both original equations
6. Rearrange equations so that LHS of each is the same
7. If RHS(1) = RHS(2), lines are coincident (infinite solutions)
   If RHS(1) $\neq$ RHS(2), lines are parallel (no solutions)

Or use Matrix -> `det` on CAS.

Solving $\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$

- Use elimination
- Generate two new equations with only two variables
- Rearrange & solve
- Substitute one variable into another equation to find another variable
- etc.