Physics

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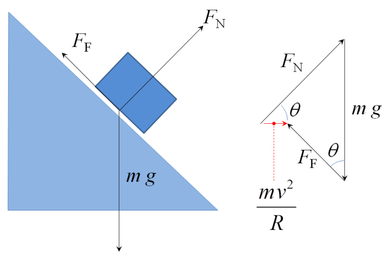
1 Motion

$\text{m/s} \times 3.6 = \text{km/h}$

Inclined planes

$F = mg \sin\theta - F_{\text{frict}} = ma$

Banked tracks



$\theta = \tan^{-1}\frac{v^2}{rg}$

ΣF always acts towards centre, but not necessarily horizontally

$\Sigma F = F_{\text{norm}} + F_{\text{g}} = \frac{mv^2}{r} = mg\tan\theta$

Design speed $v = \sqrt{gr\tan\theta}$

$n\sin\theta = mv^2 \div r, \quad n\cos\theta = mg$

Work and energy

$W = Fx = \Delta\Sigma E$ (work)

$E_K = \frac{1}{2}mv^2$ (kinetic)

$E_G = mgh$ (potential)

$\Sigma E = \frac{1}{2}mv^2 + mgh$ (energy transfer)

Horizontal circular motion

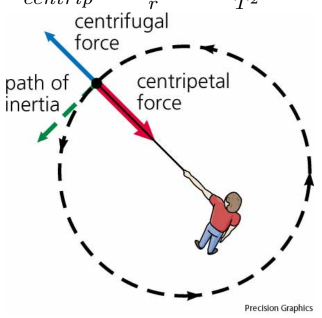
$v = \frac{2\pi r}{T}$

$f = \frac{1}{T}, \quad T = \frac{1}{f}$

$a_{centrip} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$\Sigma F, a$ towards centre, v tangential

$F_{centrip} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$



Vertical circular motion

T = tension, e.g. circular pendulum

$T + mg = \frac{mv^2}{r}$ at highest point

$T - mg = \frac{mv^2}{r}$ at lowest point

Projectile motion

- horizontal component of velocity is constant if no air resistance
- vertical component affected by gravity: $a_y = -g$

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{(vectors)}$$

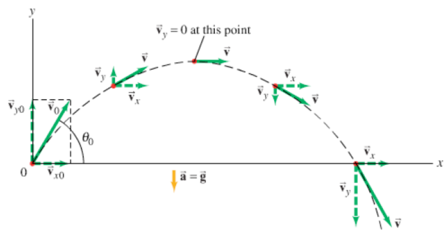
$$h = \frac{u^2 \sin\theta^2}{2g} \quad \text{(max height)}$$

$$x = ut\cos\theta \quad (\Delta x \text{ at } t)$$

$$y = ut\sin\theta - \frac{1}{2}gt^2 \quad \text{(height at } t)$$

$$t = \frac{2u\sin\theta}{g} \quad \text{(time of flight)}$$

$$d = \frac{v^2}{g}\sin\theta \quad \text{(horiz. range)}$$



Pulley-mass system

$a = \frac{m_2 g}{m_1 + m_2}$ where m_2 is suspended

$\Sigma F = m_2 g - m_1 g = \Sigma ma$ (solve)

Graphs

- Force-time: $A = \Delta\rho$
- Force-disp: $A = W$
- Force-ext: $m = k, \quad A = E_{spr}$
- Force-dist: $A = \Delta$ gpe
- Field-dist: $A = \Delta$ gpe / kg

Hooke's law

$F = -kx$

elastic potential energy $= \frac{1}{2}kx^2$

Motion equations

	no
$v = u + at$	x
$x = \frac{1}{2}(v+u)t$	a
$x = ut + \frac{1}{2}at^2$	v
$x = vt - \frac{1}{2}at^2$	u
$v^2 = u^2 + 2ax$	t

Momentum

$\rho = mv$

impulse $= \Delta\rho, \quad F\Delta t = m\Delta v$

$\Sigma m v_0 = \Sigma m v_1$ (conservation)

$\Sigma E_{K\,\text{before}} = \Sigma E_{K\,\text{after}}$ if elastic

n-body collisions: ρ of each body is independent

2 Relativity

Postulates

1. Laws of physics are constant in all intertial reference frames
2. Speed of light c is the same to all observers (Michelson-Morley)

$\therefore$ t must dilate as speed changes

Inertial reference frame $a = 0$

Proper time t_0 | **length** l_0 measured by observer in same frame as events

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t = t_0\gamma$ (t longer in moving frame)

$l = \frac{l_0}{\gamma}$ (l contracts $\parallel v$: shorter in moving frame)

$m = m_0\gamma$ (mass dilation)

$$v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

Energy and work

$E_0 = mc^2$ (rest)

$E_{total} = E_K + E_{rest} = \gamma mc^2$

$E_K = (\gamma 1)mc^2$

$W = \Delta E = \Delta mc^2$

Relativistic momentum

$$\rho = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mv = \gamma \rho_0$$

$\rho \to \infty$ as $v \to c$

$v = c$ is impossible (requires $E = \infty$)

$$v = \frac{\rho}{m\sqrt{1+\frac{p^2}{m^2c^2}}}$$

High-altitude muons

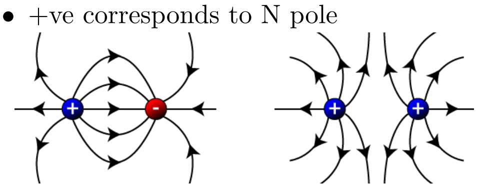
- t dilation more muons reach Earth than expected
- normal half-life $2.2\,\mu$s in stationary frame, $> 2.2\,\mu$s observed from Earth

3 Fields and power

Non-contact forces

- electric fields (dipoles & monopoles)
- magnetic fields (dipoles only)
- gravitational fields (monopoles only)

- monopoles: lines towards centre
- dipoles: field lines $+ \to -$ or N $\to$ S (or perpendicular to wire)
- closer field lines means larger force
- dot: out of page, cross: into page
- +ve corresponds to N pole



Gravity

$$F_g = G\frac{m_1 m_2}{r^2} \qquad \text{(grav. force)}$$

$$g = \frac{F_g}{m_2} = G\frac{m_1}{r^2} \qquad \text{(field of } m_1\text{)}$$

$$E_g = mg\Delta h \qquad \text{(gpe)}$$

$$W = \Delta E_g = Fx \qquad \text{(work)}$$

$$w = m(g-a) \qquad \text{(app. weight)}$$

Satellites

$$v = \sqrt{\frac{Gm_{\text{planet}}}{r}} = \sqrt{gr} = \frac{2\pi r}{T}$$

$$T = \frac{\sqrt{4\pi^2 r^2}}{GM} \qquad \text{(period)}$$

$$\sqrt[3]{\frac{GMT^2}{4\pi^2}} \qquad \text{(radius)}$$

Magnetic fields

- field strength B measured in tesla
- magnetic flux Φ measured in weber
- charge q measured in coulombs
- emf $\mathcal{E}$ measured in volts

$$F = qvB \qquad (F \text{ on moving } q)$$

$$F = IlB \qquad (F \text{ of } B \text{ on } I)$$

$$r = \frac{mv}{qB} \qquad \text{(radius of } q \text{ in } B\text{)}$$

if $B \not\perp A, \Phi \to 0$, if $B \parallel A, \Phi = 0$

Electric fields

$$F = qE \qquad (E = \text{strength})$$

$$F = k\frac{q_1 q_2}{r^2} \qquad \text{(force between } q_{1,2}\text{)}$$

$$E = k\frac{q}{r^2} \qquad \text{(field on point charge)}$$

$$E = \frac{V}{d} \qquad \text{(field between plates)}$$

$$F = BInl \qquad \text{(force on a coil)}$$

$$\Phi = B_\perp A \qquad \text{(magnetic flux)}$$

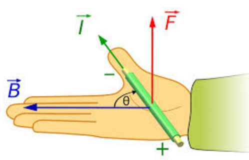
$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} \qquad \text{(induced emf)}$$

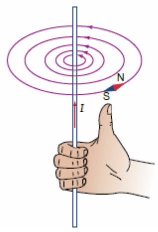
$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \qquad \text{(xfmr coil ratios)}$$

Lenz's law: I_{emf} opposes $\Delta\Phi$

Eddy currents: counter movement within a field

Right hand grip: thumb points to I (single wire) or N (solenoid / coil)





Flux-time graphs: $m \times n =$ emf

Transformers: core strengthens & focuses Φ

Particle acceleration

$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$

e- accelerated with x V is given x eV

$$W = \frac{1}{2}mv^2 = qV \qquad \text{(field or points)}$$

$$v = \sqrt{\frac{2qV}{m}} \qquad \text{(velocity of particle)}$$

Power transmission

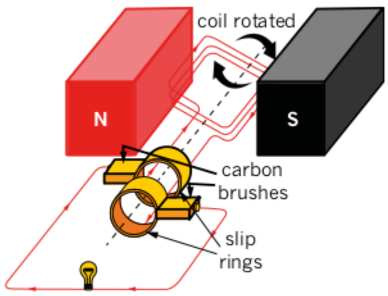
$$V_{\text{rms}} = \frac{V_{\text{p}\to\text{p}}}{\sqrt{2}}$$

$$P_{\text{loss}} = \Delta VI = I^2R = \frac{\Delta V^2}{R}$$

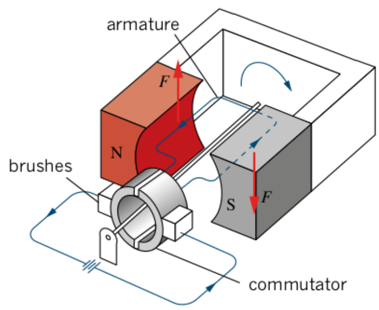
$$V_{\text{loss}} = IR$$

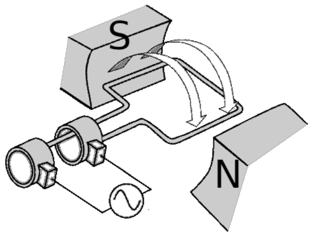
Use high-V side for correct $|V_{drop}|$

- Parallel V is constant
- Series V shared within branch



Motors





DC: split ring (two halves)

AC: slip ring (separate rings with constant contact)

4 Waves

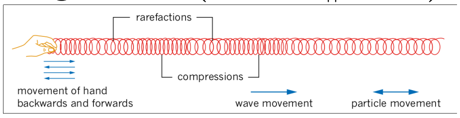
nodes: fixed on graph

amplitude: max disp. from $y = 0$

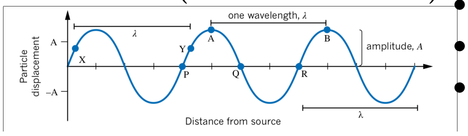
rarefactions and **compressions**

mechanical: transfer of energy without net transfer of matter

Longitudinal (motion || wave)



Transverse (motion $\perp$ wave)



$T = \frac{1}{f}$ (period: time for one cycle)

$v = f\lambda$ (speed: displacement / sec)

Doppler effect

When P_1 approaches P_2, each wave w_n has slightly less distance to travel than w_{n-1}. w_n reaches observer sooner than w_{n-1} ("apparent" λ).

Interference

Standing waves - constructive int. at resonant freq

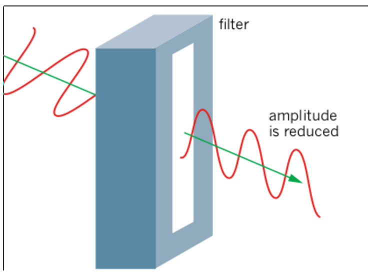
Harmonics

$\lambda = al \div n$ (λ for n^{th} harmonic)

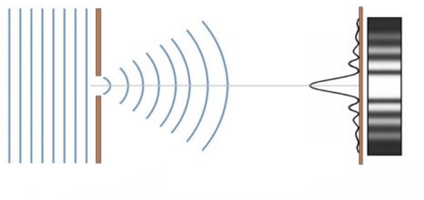
$f = nv \div al$ (f for n_{th} harmonic at length l and speed v)

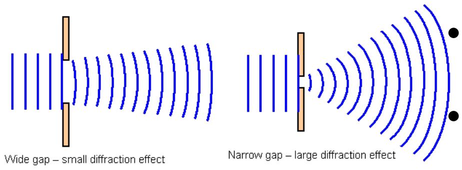
where $a = 2$ for antinodes at both ends, $a = 4$ for antinodes at one end

Polarisation



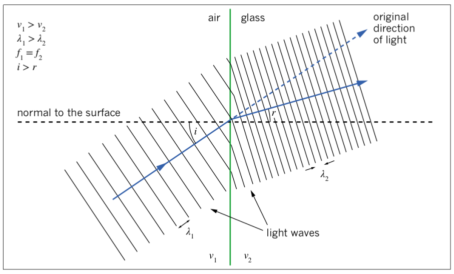
Diffraction





- $pd = |S_1P - S_2P|$ for p on screen
- Constructive: $pd = n\lambda, n \in \mathbb{Z}$
- Destructive: $pd = (n - \frac{1}{2})\lambda, n \in \mathbb{Z}$
- Fringe separation: $\Delta x = \frac{\lambda l}{d}$ where
  Δx = fringe spacing
  l = distance from slits to screen
  d = slit separation ($= S_1 - S_2$)
- significant diffraction when $\frac{\lambda}{\Delta x} \geq 1$

Refraction



When a medium changes character, energy is *reflected*, *absorbed*, and *transmitted*

angle of incidence θ_i = angle of reflection θ_r

Critical angle $\theta_c = \sin^{-}1\frac{n_2}{n_1}$

Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

5 Light and Matter

Planck's equation

$$f = \frac{c}{\lambda}, \quad E = hf = \frac{hc}{\lambda} = \rho c$$

$h = 6.63 \times 10^{-34}$ Js $= 4.14 \times 10^{-15}$ eVs

$$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$$

Force of electrons

$$F = \frac{2P_{\text{in}}}{c}$$

$$\text{photons / sec} = \frac{\text{total energy}}{\text{energy / photon}}$$

$$= \frac{P_{\text{in}}\lambda}{hc} = \frac{P_{\text{in}}}{hf}$$

Photoelectric effect

- V_{supply} does not affect photocurrent
- $V_{\text{sup}} > 0$: e- attracted to collector anode
- $V_{\text{sup}} < 0$: attracted to illuminated cathode, $I \to 0$
- v of depends on ionisation energy (shell)
- max current depends on intensity

Threshold frequency f_0

Minimum f for photoelectrons to be ejected. x-intercept of frequency vs E_K graph. if $f < f_0$, no photoelectrons are detected.

Work function ϕ

Minimum E required to release photoelectrons. Magnitude of y-intercept of frequency vs E_K graph. ϕ is determined by strength of bonding.

$\phi = hf_0$

Kinetic energy

$E_{\text{k-max}} = hf - \phi$

voltage in circuit or stopping voltage = max E_K in eV equal to x-intercept of volts vs current graph (in eV)

Stopping potential V for min I

$V = h_{\text{eV}}(f - f_0)$

De Broglie's theory

$$\lambda = \frac{h}{\rho} = \frac{h}{mv}$$

$$\rho = \frac{hf}{c} = \frac{h}{\lambda} = mv, \quad E = \rho c$$

- cannot confirm with double-slit (slit $< r_{\text{proton}}$)
- confirmed by similar e- and x-ray diff patterns

X-ray electron interaction

- e- is only stable if $mvr = n\frac{h}{2\pi}$ where $n \in \mathbb{Z}$

- rearranging this, $2\pi r = n\frac{h}{mv} = n\lambda$ (circumference)
- if $2\pi r \neq n\frac{h}{mv}$, no standing wave
- if e- = x-ray diff patterns, $E_{\text{e-}} = \frac{\rho^2}{2m} = (\frac{h}{\lambda})^2 \div 2m$
- calculating h: $\lambda = \frac{h}{\rho}$

Spectral analysis

- $\Delta E = hf = \frac{hc}{\lambda}$ between ground / excited state
- E and f of photon: $E_2 - E_1 = hf = \frac{hc}{\lambda}$
- Ionisation energy - min E required to remove e-
- EMR is absorbed/emitted when $E_{\text{K-in}} = \Delta E_{\text{shells}}$ (i.e. $\lambda = \frac{hc}{\Delta E_{\text{shells}}}$)
- No. of lines - include all possible states

Uncertainty principle

measuring location of an e- requires hitting it with a photon, but this causes ρ to be transferred to electron, moving it.

Wave-particle duaity

wave model

- cannot explain photoelectric effect
- f is irrelevant to photocurrent
- predicts delay between incidence and ejection
- speed depends on medium

particle model

- explains photoelectric effect
- rate of photoelectron release $\propto$ intensity
- no time delay - one photon releases one electron
- double slit: photons interact. interference pattern still appears when a dim light source is used so that only one photon can pass at a time
- light exerts force
- light bent by gravity
- quantised energy

6 Experimental design

Absolute uncertainty Δ
(same units as quantity)

$$\Delta(m) = \frac{\mathcal{E}(m)}{100} \cdot m$$

$$(A \pm \Delta A) + (B \pm \Delta A) = (A + B) \pm (\Delta A + \Delta B)$$

$$(A \pm \Delta A) - (B \pm \Delta A) = (A - B) \pm (\Delta A + \Delta B)$$

$$c(A \pm \Delta A) = cA \pm c\Delta A$$

Relative uncertainty $\mathcal{E}$ (unitless)

$$\mathcal{E}(m) = \frac{\Delta(m)}{m} \cdot 100$$

$$(A \pm \mathcal{E}A) \cdot (B \pm \mathcal{E}B) = (A \cdot B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A) \div (B \pm \mathcal{E}B) = (A \div B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A)^n = (A^n \pm n\mathcal{E}A)$$

$$c(A \pm \mathcal{E}A) = cA \pm \mathcal{E}A$$

Uncertainty of a measurement is $\frac{1}{2}$ the smallest division
Precision - concordance of values
Accuracy - closeness to actual value