

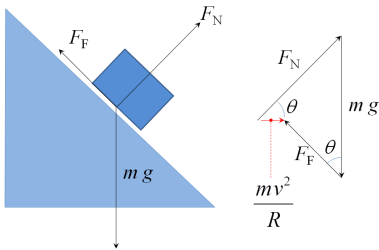
1 Motion

$m/s \times 3.6 = km/h$

Inclined planes

$F = mg \sin \theta - F_{frict} = ma$

Banked tracks



$\theta = \tan^{-1} \frac{v^2}{rg}$

ΣF always acts towards centre (horizontally)

$\Sigma F = F_{norm} + F_g = \frac{mv^2}{r} = mg \tan \theta$

Design speed $v = \sqrt{gr \tan \theta}$

$n \sin \theta = mv^2 \div r, \quad n \cos \theta = mg$

Work and energy

$W = Fs = F s \cos \theta = \Delta \Sigma E$

$E_K = \frac{1}{2}mv^2$ (kinetic)

$E_G = mgh$ (potential)

$\Sigma E = \frac{1}{2}mv^2 + mgh$ (energy transfer)

Horizontal circular motion

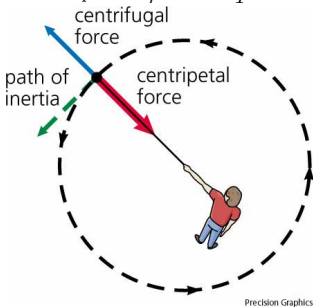
$v = \frac{2\pi r}{T}$

$f = \frac{1}{T}, \quad T = \frac{1}{f}$

$a_{centrip} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$\Sigma F, a$ towards centre, v tangential

$F_{centrip} = \frac{mv^2}{r} = \frac{4\pi^2 r m}{T^2}$



Vertical circular motion

$T =$ tension, e.g. circular pendulum

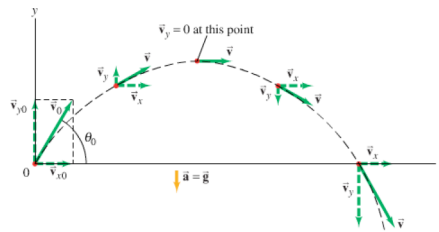
$T + mg = \frac{mv^2}{r}$ at highest point

$T - mg = \frac{mv^2}{r}$ at lowest point

$E_{K \text{ bottom}} = E_{K \text{ top}} + mgh$

Projectile motion

- v_x is constant: $v_x = \frac{s}{t}$
 - use suvat to find t from y -component
 - vertical component gravity: $a_y = -g$
- vectors
- $v = \sqrt{v_x^2 + v_y^2}$
- $h = \frac{u^2 \sin^2 \theta}{2g}$ max height
- $x = ut \cos \theta$ Δx at t
- $y = ut \sin \theta - \frac{1}{2}gt^2$ height at t
- $t = \frac{2u \sin \theta}{g}$ time of flight
- $d = \frac{v^2}{g} \sin \theta$ horiz. range



Pulley-mass system

$a = \frac{m_2 g}{m_1 + m_2}$ where m_2 is suspended

$\Sigma F = m_2 g - m_1 g = \Sigma ma$ (solve)

Graphs

- Force-time: $A = \Delta p$
- Force-disp: $A = W$
- Force-ext: $m = k, \quad A = E_{spr}$
- Force-dist: $A = \Delta gpe$
- Field-dist: $A = \Delta gpe / kg$

Hooke's law

$F = -kx$ (intercepts origin)

elastic potential energy $= \frac{1}{2}kx^2$

$x = \frac{2mg}{k}$

Vertical: $\Delta E = \frac{1}{2}kx^2 + mgh$

Motion equations

no

$v = u + at$ x

$x = \frac{1}{2}(v + u)t$ a

$x = ut + \frac{1}{2}at^2$ v

$x = vt - \frac{1}{2}at^2$ u

$v^2 = u^2 + 2ax$ t

Momentum

$p = mv$

impulse $= \Delta p, \quad F \Delta t = m \Delta v$

$\Sigma(mv_0) = (\Sigma m)v_1$ (conservation)

if elastic:

$\sum_{i=1}^n E_K(i) = \sum_{i=1}^n (\frac{1}{2}m_i v_{i0}^2) = \frac{1}{2} \sum_{i=1}^n (m_i) v_f^2$

2 Relativity

Postulates

- Laws of physics are constant in all inertial reference frames
- Speed of light c is the same to all observers (Michelson-Morley)

$\therefore t$ must dilate as speed changes

high-altitude particles: t dilation means more particles reach Earth than expected (half-life greater when obs. from Earth)

Inertial reference frame $a = 0$

Proper time t_0 | **length** l_0 measured by observer in same frame as events

Lorentz factor

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad v = c \sqrt{1 - \frac{1}{\gamma^2}}$

$t = t_0 \gamma$ (t longer in moving frame)

$l = \frac{l_0}{\gamma}$ (l contracts $\parallel v$: shorter in moving frame)

$m = m_0 \gamma$ (mass dilation)

Energy and work

$E_{rest} = mc^2, \quad E_K = (\gamma - 1)mc^2$

$E_{total} = E_K + E_{rest} = \gamma mc^2$

$W = \Delta E = \Delta mc^2 = (\gamma - 1)m_{rest}c^2$

Relativistic momentum

$$\rho = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mv = \gamma \rho_0$$

$\rho \rightarrow \infty$ as $v \rightarrow c$

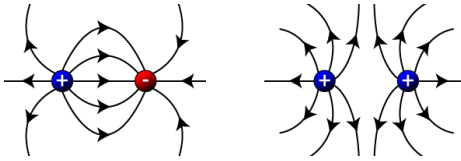
$v = c$ is impossible (requires $E = \infty$)

$$v = \frac{\rho}{m\sqrt{1 + \frac{\rho^2}{m^2 c^2}}}$$

3 Fields and power

Non-contact forces

- electric (dipoles & monopoles)
- magnetic (dipoles only)
- gravitational (monopoles only, $F_g = 0$ at mid, attractive only)
- monopoles: lines towards centre
- dipoles: field lines $+$ \rightarrow $-$ or $N \rightarrow S$ (two magnets) or $\rightarrow N$ (single)
- closer field lines means larger force
- dot: out of page, cross: into page
- +ve corresponds to N pole



Gravity

$$F_g = G \frac{m_1 m_2}{r^2} \quad (\text{grav. force})$$

$$g = \frac{F_g}{m_2} = G \frac{m_1}{r^2} \quad (\text{field of } m_1)$$

$$E_g = mg\Delta h \quad (\text{gpe})$$

$$W = \Delta E_g = Fx \quad (\text{work})$$

$$w = m(g - a) \quad (\text{app. weight})$$

Satellites

$$v = \sqrt{\frac{GM}{r}} = \sqrt{gr} = \frac{2\pi r}{T}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} \quad (\text{period})$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \quad (\text{radius})$$

Magnetic fields

- field strength B measured in tesla
- magnetic flux Φ measured in weber
- charge q measured in coulombs
- emf \mathcal{E} measured in volts

$$F = qvB \quad (F \text{ on moving } q)$$

$$F = IlB \quad (F \text{ of } B \text{ on } I)$$

$$B = \frac{mv}{qr} \quad (\text{field strength on } e^-)$$

$$r = \frac{mv}{qB} \quad (\text{radius of } q \text{ in } B)$$

if $B \perp A, \Phi \rightarrow 0$, if $B \parallel A, \Phi = 0$

Electric fields

$$F = qE (= ma) \quad (\text{strength})$$

$$F = k \frac{q_1 q_2}{r^2} \quad (\text{force between } q_1, 2)$$

$$E = k \frac{q}{r^2} \quad (\text{field on point charge})$$

$$E = \frac{V}{d} \quad (\text{field between plates})$$

$$F = BIl \quad (\text{force on a coil})$$

$$\Phi = B_{\perp} A \quad (\text{magnetic flux})$$

$$\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} = Blv \quad (\text{induced emf})$$

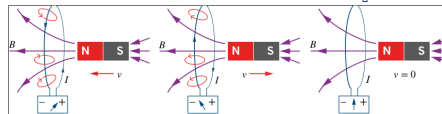
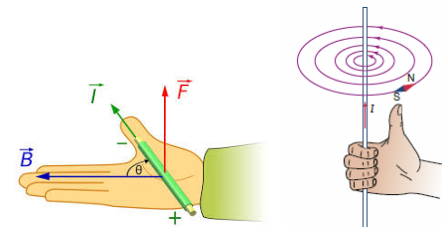
$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \quad (\text{xfmr coil ratios})$$

Lenz's law: I_{emf} opposes $\Delta \Phi$

(emf creates I with associated field that opposes $\Delta \Phi$)

Eddy currents: counter movement within a field

Right hand grip: thumb points to I (single wire) or N (solenoid / coil)



Flux-time graphs: $m \times n = \text{emf}$. If f increases, ampl. & f of \mathcal{E} increase

Xfmr core strengthens & focuses Φ

Particle acceleration

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

e^- accelerated with $x \text{ V}$ is given $x \text{ eV}$

$$W = \frac{1}{2}mv^2 = qV \quad (\text{field or points})$$

$$v = \sqrt{\frac{2qV}{m}} \quad (\text{velocity of particle})$$

Circular path: $F \perp B \perp v$

Power transmission

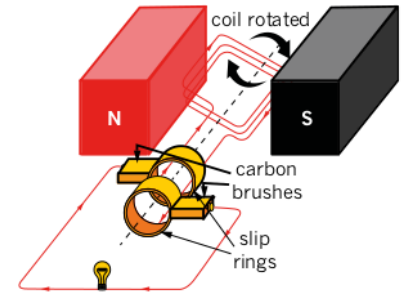
$$V_{\text{rms}} = \frac{V_{\text{p-p}}}{\sqrt{2}}$$

$$P_{\text{loss}} = \Delta VI = I^2 R = \frac{\Delta V^2}{R}$$

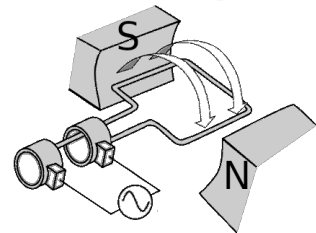
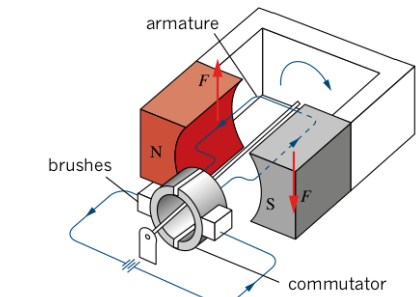
$$V_{\text{loss}} = IR$$

Use high- V side for correct $|V_{\text{drop}}|$

- Parallel V is constant
- Series V shared within branch



Motors



Force on current-carrying wire, not copper

$F = 0$ for front back of coil (parallel)

Any angle > 0 will produce force

DC: split ring (two halves)

AC: slip ring (separate rings with constant contact)

4 Waves

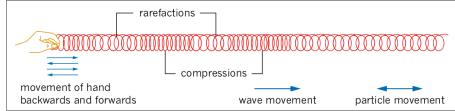
nodes: fixed on graph

amplitude: max disp. from $y = 0$

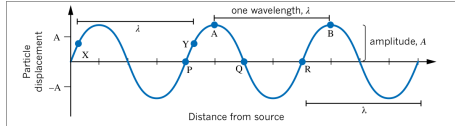
rarefactions and compressions

mechanical: transfer of energy without net transfer of matter

Longitudinal (motion || wave)



Transverse (motion ⊥ wave)



$$T = \frac{1}{f} \quad (\text{period: time for one cycle})$$

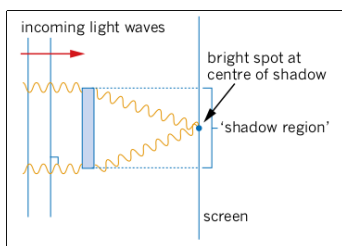
$$v = f\lambda \quad (\text{speed: displacement / sec})$$

$$f = \frac{c}{\lambda} \quad (\text{for } v = c)$$

Doppler effect

When P_1 approaches P_2 , each wave w_n has slightly less distance to travel than w_{n-1} . w_n reaches observer sooner than w_{n-1} ("apparent" λ).

Interference



Poisson's spot supports wave theory (circular diffraction)

Standing waves - constructive int. at resonant freq. Rebound from ends.

Coherent - identical frequency, phase, direction (ie strong directional). e.g. laser

Incoherent - e.g. incandescent/LED

Harmonics

1st harmonic = fundamental

for nodes at both ends:

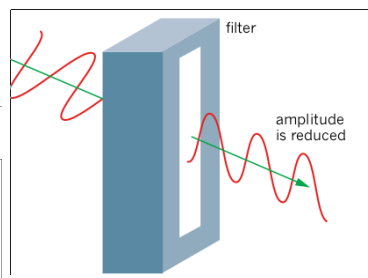
$$\lambda = 2l \div n \quad f = nv \div 2l$$

for node at one end (n is odd):

$$\lambda = 4l \div n \quad f = nv \div 4l$$

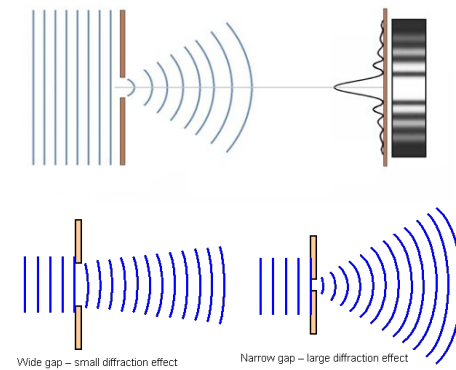
alternatively, $\lambda = \frac{4l}{2n-1}$ where $n \in \mathbb{Z}$ and $n+1$ is the next possible harmonic

Polarisation



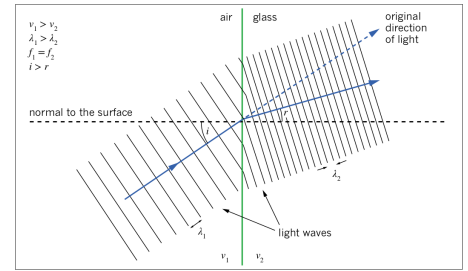
Transverse only. Reduces total A .

Diffraction



- Constructive: $pd = n\lambda, n \in \mathbb{Z}$
- Destructive: $pd = (n - \frac{1}{2})\lambda, n \in \mathbb{Z}$
- Path difference: $\Delta x = \frac{\lambda}{d}$ where l = distance from source to observer d = separation between each wave source (e.g. slit) = $S_1 - S_2$
- diffraction $\propto \frac{\lambda}{d}$
- significant diffraction when $\frac{\lambda}{\Delta x} \geq 1$
- diffraction creates distortion (electron > optical microscopes)

Refraction



When a medium changes character, light is *reflected*, *absorbed*, and *transmitted*

angle of incidence θ_i = angle of reflection θ_r

$$\text{Critical angle } \theta_c = \sin^{-1} \frac{n_2}{n_1}$$

Snell's law $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$$v_1 \div v_2 = \sin \theta_1 \div \sin \theta_2$$

$$n_1 v_1 = n_2 v_2$$

$$n = \frac{c}{v}$$

5 Light and Matter

Planck's equation

$$E = hf = \frac{hc}{\lambda} = \rho c = qV$$

$$h = 6.63 \times 10^{-34} \text{ Js} = 4.14 \times 10^{-15} \text{ eVs}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

De Broglie's theory

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2W}{m}}}$$

$$\rho = \frac{hf}{c} = \frac{h}{\lambda} = mv, \quad E = \rho c$$

$$v = \sqrt{2E_K \div m}$$

- cannot confirm with double-slit (slit $< r_{\text{proton}}$)
- confirmed by e- and x-ray patterns

Force of electrons

$$F = \frac{2P_{\text{in}}}{c}$$

$$\text{photons / sec} = \frac{\text{total energy}}{\text{energy / photon}}$$

$$= \frac{P_{\text{in}}\lambda}{hc} = \frac{P_{\text{in}}}{hf}$$

X-ray electron interaction

- e- stable if $mvr = n \frac{h}{2\pi}$ where $n \in \mathbb{Z}$ and r is radius of orbit
- $\therefore 2\pi r = n \frac{h}{mv} = n\lambda$ (circumference)
- if $2\pi r \neq n \frac{h}{mv}$, no standing wave
- if e- = x-ray diff patterns, $E_{e-} = \frac{p^2}{2m} = (\frac{h}{\lambda})^2 \div 2m$

Photoelectric effect

- V_{supply} does not affect photocurrent
- $V_{\text{sup}} > 0$: attracted to +ve
- $V_{\text{sup}} < 0$: attracted to -ve, $I \rightarrow 0$
- v of e- depends on shell
- max I (not V) depends on intensity

Threshold frequency f_0

min f for photoelectron release. if $f < f_0$, no photoelectrons.

Work function $\phi = hf_0$

min E for photoelectron release. determined by strength of bonding. Units: eV or J.

Kinetic energy $E_K = hf - \phi = qV_0$

$V_0 = E_K$ in eV
dashed line below $E_K = 0$

Stopping potential V_0 for min I

$$V_0 = h_{\text{eV}}(f - f_0)$$

Opposes induced photocurrent

Graph features

	m	x -int	y -int
$f \cdot E_K$	h	f_0	$-\phi$
$V \cdot I$		V_0	intensity
$f \cdot V$	$\frac{h}{q}$	f_0	$\frac{-\phi}{q}$

Spectral analysis

- $\Delta E = hf = \frac{hc}{\lambda}$ between ground / excited state
- E and f of photon: $E_2 - E_1 = hf = \frac{hc}{\lambda}$
- Ionisation energy - min E required to remove e-
- EMR is absorbed/emitted when $E_{K\text{-in}} = \Delta E_{\text{shells}}$ (i.e. $\lambda = \frac{hc}{\Delta E_{\text{shells}}}$)
- No. of lines - include all possible states

Uncertainty principle

measuring location of an e- requires hitting it with a photon, but this causes p to be transferred to electron, moving it.

Wave-particle duality

wave model

- cannot explain photoelectric effect
- f is irrelevant to photocurrent
- predicts delay between incidence and ejection
- speed depends on medium
- supported by bright spot in centre
- $\lambda = \frac{hc}{E}$

particle model

- explains photoelectric effect
- rate of photoelectron release \propto intensity
- no time delay - one photon releases one electron
- double slit: photons interact. interference pattern still appears when a dim light source is used so that only one photon can pass at a time
- light exerts force

- light bent by gravity
- quantised energy
- $\lambda = \frac{h}{p}$

6 Experimental design

Absolute uncertainty Δ
(same units as quantity)

$$\Delta(m) = \frac{\mathcal{E}(m)}{100} \cdot m$$

$$(A \pm \Delta A) + (B \pm \Delta B) = (A+B) \pm (\Delta A + \Delta B)$$

$$(A \pm \Delta A) - (B \pm \Delta B) = (A-B) \pm (\Delta A + \Delta B)$$

$$c(A \pm \Delta A) = cA \pm c\Delta A$$

Relative uncertainty \mathcal{E} (unitless)

$$\mathcal{E}(m) = \frac{\Delta(m)}{m} \cdot 100$$

$$(A \pm \mathcal{E}A) \cdot (B \pm \mathcal{E}B) = (A \cdot B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A) \div (B \pm \mathcal{E}B) = (A \div B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A)^n = (A^n \pm n\mathcal{E}A)$$

$$c(A \pm \mathcal{E}A) = cA \pm \mathcal{E}A$$

Uncertainty of a measurement is $\frac{1}{2}$ the smallest division

Precision - concordance of values

Accuracy - closeness to actual value

Random errors - unpredictable, reduced by more tests

Systematic errors - not reduced by more tests

Uncertainty - margin of potential error

Error - actual difference

Hypothesis - can be tested experimentally

Model - evidence-based but indirect representation

