Methods - Calculus

Average rate of change

$$m \text{ of } x \in [a,b] = \frac{f(b)-f(a)}{b-a} = \frac{dy}{dx}$$

Average rate of change between $x = [a,b]$ given two points $P(a, f(a))$ and $Q(b, f(b))$ is the gradient m of line $\overleftrightarrow{PQ}$

On CAS: (Action|Interactive) -> Calculation -> Diff -> $f(x)$ or $y = \ldots$

Instantaneous rate of change

Secant - line passing through two points on a curve
Chord - line segment joining two points on a curve

Estimated by using two given points on each side of the concerned point. Evaluate as in average rate of change.

Limits & continuity

Limit theorems

1. For constant function $f(x) = k$, $\lim_{x\to a} f(x) = k$
2. $\lim_{x\to a}(f(x) \pm g(x)) = F \pm G$
3. $\lim_{x\to a}(f(x) \times g(x)) = F \times G$
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{F}{G}, G \neq 0$

A function is continuous if $L^- = L^+ = f(x)$ for all values of x.

First principles derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Tangents & gradients

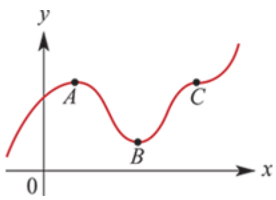
Tangent line - defined by $y = mx + c$ where $m = \frac{dy}{dx}$
Normal line - $\perp$ tangent ($m_{\text{tan}} \cdot m_{\text{norm}} = -1$)
Secant = $\frac{f(x+h)-f(x)}{h}$

Solving on CAS

In main: type function. Interactive -> Calculation -> Line -> (Normal | Tan line)
In graph: define function. Analysis -> Sketch -> (Normal | Tan line). Type x value to solve for a point. Return to show equation for line.

Stationary points

Stationary where $m = 0$.
Find derivative, solve for $\frac{dy}{dx} = 0$



Local maximum at point A
- $f'(x) > 0$ left of A - $f'(x) < 0$ right of A

Local minimum at point B
- $f'(x) < 0$ left of B - $f'(x) > 0$ right of B

Stationary point of inflection at C

Function derivatives

$f(x)$	$f'(x)$
x^n	nx^{n-1}
kx^n	knx^{n-1}
$g(x) + h(x)$	$g'(x) + h'(x)$
c	0
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
$f \circ g$	$\frac{dy}{du} \cdot \frac{du}{dx}$