1 Dynamics

Resolution of forces

Resultant force is sum of force vectors

In angle-magnitude form

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos\theta$ Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

In $\boldsymbol{i}$—$\boldsymbol{j}$ form

Vector of a N at θ to x axis is equal to $a\cos\theta\boldsymbol{i} + a\sin\theta\boldsymbol{j}$. Convert all force vectors then add.

To find angle of an $a\boldsymbol{i} + b\boldsymbol{j}$ vector, use $\theta = \tan^{-1}\frac{b}{a}$

Resolving in a given direction

The resolved part of a force P at angle θ is has magnitude $P\cos\theta$

To convert force $||\vec{OA}$ to angle-magnitude form, find component $\perp \vec{OA}$ then:

$$|\boldsymbol{r}| = \sqrt{\left(||\vec{OA}\right)^2 + \left(\perp \vec{OA}\right)^2}$$

$$\theta = \tan^{-1}\frac{\perp \vec{OA}}{||\vec{OA}}$$

Newton's laws

1. Velocity is constant without ΣF
2. $\frac{d}{dt}\rho \propto \Sigma F \implies \boldsymbol{F} = m\boldsymbol{a}$
3. Equal and opposite forces

Weight

A mass of m kg has force of mg acting on it

Momentum ρ

$$\rho = mv \qquad \text{(units kg m/s or Ns)}$$

Reaction force R

- With no vertical velocity, $R = mg$
- With vertical acceleration, $|R| = m|a| - mg$
- With force F at angle θ, then $R = mg - F\sin\theta$

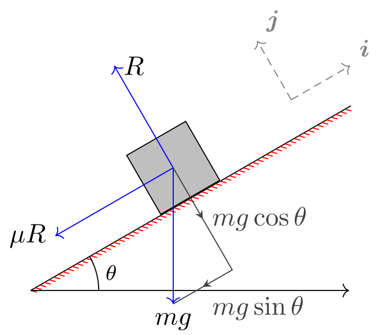
Friction

$$F_R = \mu R \qquad \text{(friction coefficient)}$$

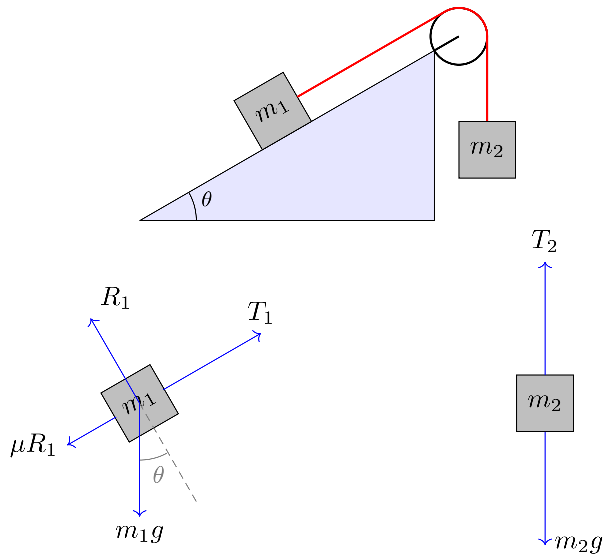
Inclined planes

$$\boldsymbol{F} = |\boldsymbol{F}| \cos\theta \boldsymbol{i} + |\boldsymbol{F}| \sin\theta \boldsymbol{j}$$

- Normal force R is at right angles to plane
- Let direction up the plane be $\boldsymbol{i}$ and perpendicular to plane $\boldsymbol{j}$



Connected particles



- **Suspended pulley:** tension in both sections of rope are equal
  $|a| = g\frac{m_1 - m_2}{m_1 + m_2}$ where m_1 accelerates down
  With tension:

$$\begin{cases} m_1 g - T = m_1 a \\ T - m_2 g = m_2 a \end{cases} \implies m_1 g - m_2 g = m_1 a + m_2 a$$

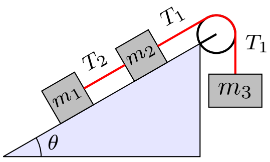
- **String pulling mass on inclined pane:** Resolve parallel to plane

$$T - mg \sin\theta = ma$$

- **Linear connection:** find acceleration of system first
- **Pulley on right angle:** $a = \frac{m_2 g}{m_1 + m_2}$ where m_2 is suspended (frictionless on both surfaces)

- **Pulley on edge of incline:** find downwards force W_2 and components of mass on plane

In this example, note $T_1 \neq T_2$:



Equilibrium

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \qquad \text{(Lami's theorem)}$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta \qquad \text{(cosine rule)}$$

Three methods:

1. Lami's theorem (sine rule)
2. Triangle of forces (cosine rule)
3. Resolution of forces ($\Sigma F = 0$ - simultaneous)

On CAS

To verify: Geometry tab, then select points with normal cursor. Click right arrow at end of toolbar and input point, then lock known constants.

Variable forces (DEs)

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$