# Physics

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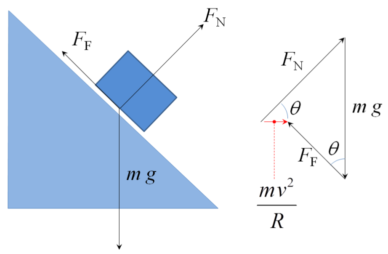
## 1 Motion

m/s $\times 3.6 =$ km/h

### Inclined planes

$F = mg\sin\theta - F_{frict} = ma$

### Banked tracks




$$\theta = \tan^{-1}\frac{v^2}{rg}$$

$\Sigma F$ always acts towards centre, but not necessarily horizontally

$\Sigma F = F_{\text{norm}} + F_{\text{g}} = \frac{mv^2}{r} = mg\tan\theta$

Design speed $v = \sqrt{gr\tan\theta}$

### Work and energy

$W = Fx = \Delta\Sigma E$ (work)

$E_K = \frac{1}{2}mv^2$ (kinetic)

$E_G = mgh$ (potential)

$\Sigma E = \frac{1}{2}mv^2 + mgh$ (energy transfer)

### Horizontal circular motion

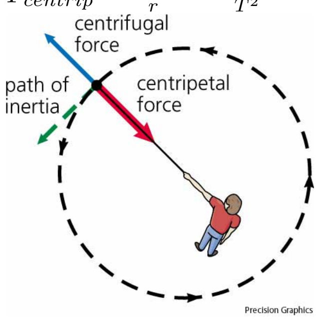
$v = \frac{2\pi r}{T}$

$f = \frac{1}{T}, \quad T = \frac{1}{f}$

$a_{centrip} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$\Sigma F, a$ towards centre, $v$ tangential

$F_{centrip} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$




### Vertical circular motion

$T =$ tension, e.g. circular pendulum

$T + mg = \frac{mv^2}{r}$ at highest point

$T - mg = \frac{mv^2}{r}$ at lowest point

### Projectile motion

- horizontal component of velocity is constant if no air resistance
- vertical component affected by gravity: $a_y = -g$

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{(vectors)}$$

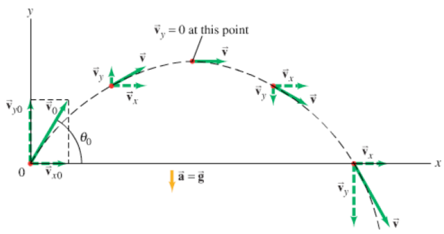
$$h = \frac{u^2\sin\theta^2}{2g} \quad \text{(max height)}$$

$$x = ut\cos\theta \quad (\Delta x \text{ at } t)$$

$$y = ut\sin\theta - \frac{1}{2}gt^2 \quad \text{(height at } t)$$

$$t = \frac{2u\sin\theta}{g} \quad \text{(time of flight)}$$

$$d = \frac{v^2}{g}\sin\theta \quad \text{(horiz. range)}$$




### Pulley-mass system

$a = \frac{m_2 g}{m_1 + m_2}$ where $m_2$ is suspended

$\Sigma F = m_2 g - m_1 g = \Sigma ma$ (solve)

### Graphs

- Force-time: $A = \Delta\rho$
- Force-disp: $A = W$
- Force-ext: $m = k, \quad A = E_{spr}$
- Force-dist: $A = \Delta$ gpe
- Field-dist: $A = \Delta$ gpe / kg

### Hooke's law

$F = -kx$

$E_{elastic} = \frac{1}{2}kx^2$

### Motion equations

| | |
|---|---|
| $v = u + at$ | $x$ |
| $x = \frac{1}{2}(v+u)t$ | $a$ |
| $x = ut + \frac{1}{2}at^2$ | $v$ |
| $x = vt - \frac{1}{2}at^2$ | $u$ |
| $v^2 = u^2 + 2ax$ | $t$ |

### Momentum

$\rho = mv$

impulse $= \Delta\rho, \quad F\Delta t = m\Delta v$

$\Sigma mv_0 = \Sigma mv_1$ (conservation)

$\Sigma E_{K\text{ before}} = \Sigma E_{K\text{ after}}$ if elastic

$n$-body collisions: $\rho$ of each body is independent

## 2 Relativity

### Postulates

1. Laws of physics are constant in all intertial reference frames

2. Speed of light $c$ is the same to all observers (Michelson-Morley)

$\therefore, t$ must dilate as speed changes

**Inertial reference frame** - $a = 0$

**Proper time** $t_0$ | **length** $l_0$ - measured by observer in same frame as events

### Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t = t_0\gamma$ ($t$ longer in moving frame)

$l = \frac{l_0}{\gamma}$ ($l$ contracts $\parallel v$: shorter in moving frame)

$m = m_0\gamma$ (mass dilation)

$$v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

### Energy and work

$E_0 = mc^2$ (rest)

$E_{total} = E_K + E_{rest} = \gamma mc^2$

$E_K = (\gamma - 1)mc^2$

$W = \Delta E = \Delta mc^2$

### Relativistic momentum

$$\rho = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m v = \gamma \rho_0$$

$\rho \to \infty$ as $v \to c$

$v = c$ is impossible (requires $E = \infty$)

$$v = \frac{\rho}{m\sqrt{1+\frac{p^2}{m^2c^2}}}$$

### High-altitude muons

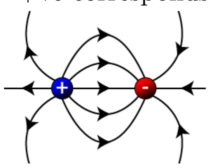
- $t$ dilation - more muons reach Earth than expected
- normal half-life $2.2\,\mu$s in stationary frame, $> 2.2\,\mu$s observed from Earth

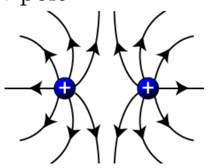
## 3 Fields and power

### Non-contact forces

- electric fields (dipoles & monopoles)
- magnetic fields (dipoles only)
- gravitational fields (monopoles only)

- monopoles: lines towards centre
- dipoles: field lines $+ \to -$ or N $\to$ S (or perpendicular to wire)
- closer field lines means larger force
- dot: out of page, cross: into page
- +ve corresponds to N pole





### Gravity

$$F_g = G\frac{m_1 m_2}{r^2} \qquad \text{(grav. force)}$$

$$g = \frac{F_g}{m_2} = G\frac{m_1}{r^2} \qquad \text{(field of } m_1\text{)}$$

$$E_g = mg\Delta h \qquad \text{(gpe)}$$

$$W = \Delta E_g = Fx \qquad \text{(work)}$$

$$w = m(g-a) \qquad \text{(app. weight)}$$

### Satellites

$$v = \sqrt{\frac{Gm_{\text{planet}}}{r}} = \sqrt{gr} = \frac{2\pi r}{T}$$

$$T = \frac{\sqrt{4\pi^2 r^2}}{GM} \qquad \text{(period)}$$

$$\sqrt[3]{\frac{GMT^2}{4\pi^2}} \qquad \text{(radius)}$$

### Magnetic fields

- field strength $B$ measured in tesla
- magnetic flux $\Phi$ measured in weber
- charge $q$ measured in coulombs
- emf $\mathcal{E}$ measured in volts

$$F = qvB \qquad (F \text{ on moving } q)$$

$$F = IlB \qquad (F \text{ of } B \text{ on } I)$$

$$r = \frac{mv}{qB} \qquad \text{(radius of } q \text{ in } B\text{)}$$

if $B \not\perp A, \Phi \to 0$ , if $B \parallel A, \Phi = 0$

### Electric fields

$$F = qE \qquad (E = \text{strength})$$

$$F = k\frac{q_1 q_2}{r^2} \qquad \text{(force between } q_{1,2}\text{)}$$

$$E = k\frac{q}{r^2} \qquad \text{(field on point charge)}$$

$$E = \frac{V}{d} \qquad \text{(field between plates)}$$

$$F = BInl \qquad \text{(force on a coil)}$$

$$\Phi = B_\perp A \qquad \text{(magnetic flux)}$$

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} \qquad \text{(induced emf)}$$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \qquad \text{(xfmr coil ratios)}$$

**Lenz's law:** $I_{\text{emf}}$ opposes $\Delta\Phi$

**Eddy currents:** counter movement within a field

**Right hand grip:** thumb points to $I$ (single wire) or N (solenoid / coil)

**Right hand slap:** $B \perp I \perp F$

**Flux-time graphs:** $m \times n =$ emf

**Transformers:** core strengthens & focuses $\Phi$

### Particle acceleration

$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$

e- accelerated with $x$ V is given $x$ eV

$$W = \frac{1}{2}mv^2 = qV \qquad \text{(field or points)}$$

$$v = \sqrt{\frac{2qV}{m}} \qquad \text{(velocity of particle)}$$

### Power transmission

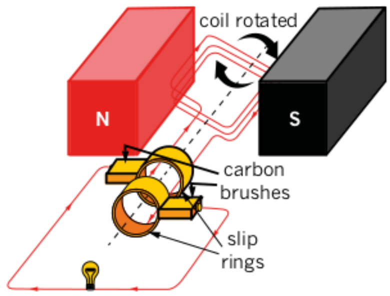
$$V_{\text{rms}} = \frac{V_{\text{p}\to\text{p}}}{\sqrt{2}}$$

$P_{\text{loss}} = \Delta V I = I^2 R = \frac{\Delta V^2}{R}$

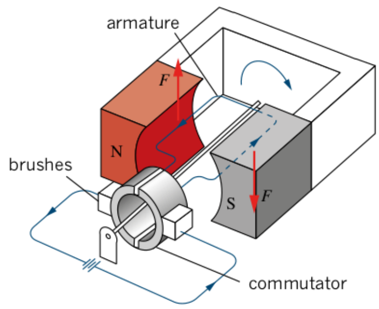
$V_{\text{loss}} = IR$

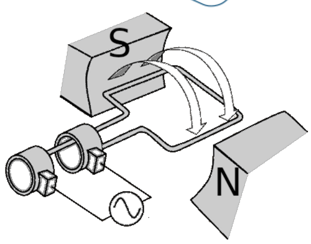
Use high-$V$ side for correct $|V_{drop}|$

- Parallel - $V$ is constant
- Series - $V$ shared within branch




### Motors




**DC:** split ring (two halves)

**AC:** slip ring (separate rings with constant contact)