Physics

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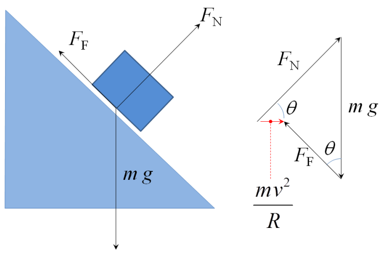
1 Motion

Inclined planes

$F = mg\sin\theta - F_{frict} = ma$

Banked tracks



$\theta = \tan^{-1}\frac{v^2}{rg}$ (also for objects on string)

ΣF always acts towards centre, but not necessarily horizontally

$\Sigma F = \frac{mv^2}{r} = mg\tan\theta$

Design speed $v = \sqrt{gr\tan\theta}$

Work and energy

$W = Fx = \Delta\Sigma E$ (work)

$E_K = \frac{1}{2}mv^2$ (kinetic)

$E_G = mgh$ (potential)

$\Sigma E = \frac{1}{2}mv^2 + mgh$ (energy transfer)

Horizontal motion

m/s $\times 3.6 =$ km/h

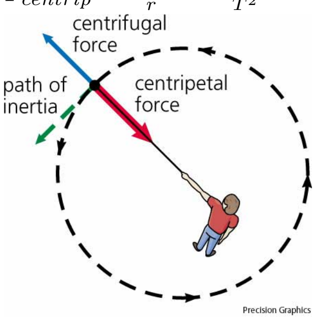
$v = \frac{2\pi r}{T}$

$f = \frac{1}{T}, \quad T = \frac{1}{f}$

$a_{centrip} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

ΣF towards centre, v tangential

$F_{centrip} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$



Vertical circular motion

$T =$ tension, e.g. circular pendulum

$T + mg = \frac{mv^2}{r}$ at highest point

$T - mg = \frac{mv^2}{r}$ at lowest point

Projectile motion

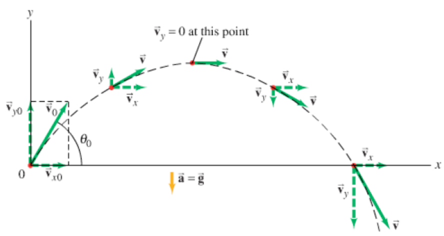
- horizontal component of velocity is constant if no air resistance
- vertical component affected by gravity: $a_y = -g$

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{(vectors)}$$

$$h = \frac{u^2\sin\theta^2}{2g} \quad \text{(max height)}$$

$$y = ut\sin\theta - \frac{1}{2}gt^2 \quad \text{(time of flight)}$$

$$d = \frac{v^2}{g}\sin\theta \quad \text{(horiz. range)}$$



Pulley-mass system

$a = \frac{m_2 g}{m_1 + m_2}$ where m_2 is suspended

$\Sigma F = m_2 g - m_1 g = \Sigma ma$ (solve)

Graphs

- Force-time: $A = \Delta\rho$
- Force-disp: $A = W$
- Force-ext: $m = k, \quad A = E_{spr}$
- Force-dist: $A = \Delta$ gpe
- Field-dist: $A = \Delta$ gpe / kg

Hooke's law

$F = -kx$

$E_{elastic} = \frac{1}{2}kx^2$

Motion equations

$v = u + at$	x
$x = \frac{1}{2}(v+u)t$	a
$x = ut + \frac{1}{2}at^2$	v
$x = vt - \frac{1}{2}at^2$	u
$v^2 = u^2 + 2ax$	t

Momentum

$\rho = mv$

impulse $= \Delta\rho, \quad F\Delta t = m\Delta v$

Momentum is conserved.

$\Sigma E_{K\,\text{before}} = \Sigma E_{K\,\text{after}}$ if elastic

n-body collisions: ρ of each body is independent

2 Relativity

Postulates

1. Laws of physics are constant in all intertial reference frames
2. Speed of light c is the same to all observers (Michelson-Morley)

$\therefore, t$ must dilate as speed changes

Inertial reference frame - $a = 0$

Proper time t_0 | **length** l_0 - measured by observer in same frame as events

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t = t_0\gamma$ (t longer in moving frame)

$l = \frac{l_0}{\gamma}$ (l contracts $\parallel v$: shorter in moving frame)

$m = m_0\gamma$ (mass dilation)

$$v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

Energy and work

$E_0 = mc^2$ (rest)

$E_{total} = E_K + E_{rest} = \gamma mc^2$

$E_K = (\gamma - 1)mc^2$

$W = \Delta E = \Delta mc^2$

Relativistic momentum

$$\rho = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mv = \gamma \rho_0$$

$\rho \to \infty$ as $v \to c$

$v = c$ is impossible (requires $E = \infty$)

$$v = \frac{\rho}{m\sqrt{1+\frac{p^2}{m^2c^2}}}$$

Fusion and fission

$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$

e- accelerated with x V is given x eV

High-altitude muons

- t dilation - more muons reach Earth than expected
- normal half-life is $2.2\,\mu s$ in stationary frame
- at $v \approx c$, muons observed from Earth have halflife $> 2.2\,\mu s$
- slower time - more time to travel, so muons reach surface

3 Fields and power

Non-contact forces

- electric fields (dipoles & monopoles)
- magnetic fields (dipoles only)
- gravitational fields (monopoles only)

- monopoles: lines towards centre
- dipoles: field lines $+ \to -$ or N $\to$ S (or perpendicular to wire)
- closer field lines means larger force
- dot means out of page, cross means into page
- +ve corresponds to N pole

Gravity

$$F_g = G\frac{m_1 m_2}{r^2} \qquad \text{(grav. force)}$$

$$g = \frac{F_g}{m} = G\frac{M_{\text{planet}}}{r^2} \qquad \text{(grav. acc.)}$$

$$E_g = mg\Delta h \qquad \text{(gpe)}$$

$$W = \Delta E_g = Fx \qquad \text{(work)}$$

$$w = m(g-a) \qquad \text{(app. weight)}$$

Satellites

$$v = \sqrt{\frac{GM}{r}} = \sqrt{gr} = \frac{2\pi r}{T}$$

$$T = \frac{\sqrt{4\pi^2 r^2}}{GM} \qquad \text{(period)}$$

$$\sqrt[3]{\frac{GMT^2}{4\pi^2}} \qquad \text{(radius)}$$

Magnetic fields

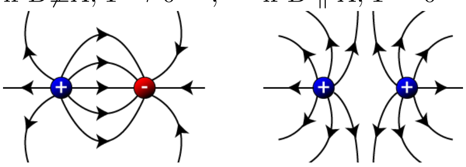
- field strength B measured in tesla
- magnetic flux Φ measured in weber
- charge q measured in coulombs
- emf $\mathcal{E}$ measured in volts

$$\frac{E_1}{E_2} = \frac{r_1}{r_2}^2$$

$$F = qvB$$

(force on moving charged particles)

if $B \not\parallel A, \Phi \to 0$, if $B \parallel A, \Phi = 0$



Electric fields

$$F = qE \qquad (E = \text{strength})$$

$$W = q_{\text{point}}\Delta V \qquad \text{(in field or points)}$$

$$F = k\frac{q_1 q_2}{r^2} \qquad \text{(force between } q_{1,2})$$

$$E = k\frac{Q}{r^2} \qquad (r = ||EQ||)$$

$$F = BInl \qquad \text{(force on a coil)}$$

$$\Phi = B_\perp A \qquad \text{(magnetic flux)}$$

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} \qquad \text{(induced emf)}$$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \qquad \text{(xfmr coil ratios)}$$

Lenz's law: "$-n$" in Faraday - emf opposes $\Delta\Phi$

Eddy currents: counter movement within a field

Right hand grip: thumb points to north or I

Right hand slap: field, current, force are $\perp$

Flux-time graphs: gradient $\times n$ = emf

Transformers: core strengthens & focuses Φ

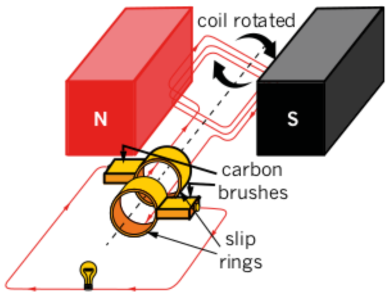
Power transmission

$$V_{\text{rms}} = \frac{V_{\text{p}\to\text{p}}}{\sqrt{2}}$$

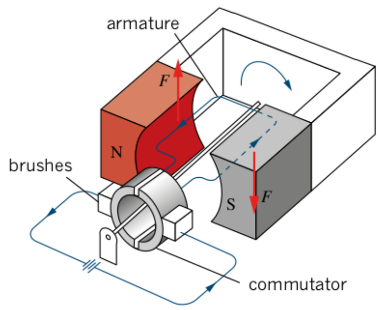
$$P_{\text{loss}} = \Delta VI = I^2R = \frac{\Delta V^2}{R}$$

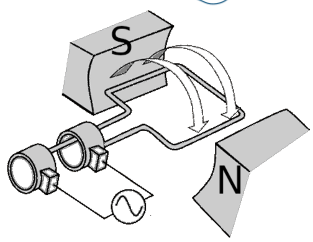
Use high-V side for correct $|V_{drop}|$

- Parallel - voltage is constant
- Series - voltage is shared within branch



Motors





DC: split ring (two halves)

AC: slip ring (separate rings with constant contact)