Calculus

Average rate of change

$$m \text{ of } x \in [a,b] = \frac{f(b)-f(a)}{b-a} = \frac{dy}{dx}$$

On CAS: Action → Calculation → `diff`

Instantaneous rate of change

Secant - line passing through two points on a curve
Chord - line segment joining two points on a curve

Limit theorems

1. For constant function $f(x)=k$, $\lim_{x\to a} f(x) = k$
2. $\lim_{x\to a}(f(x) \pm g(x)) = F \pm G$
3. $\lim_{x\to a}(f(x) \times g(x)) = F \times G$
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{F}{G}, G \neq 0$

A function is continuous if $L^- = L^+ = f(x)$ for all values of x.

First principles derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Not differentiable at:

- discontinuous points
- sharp point/cusp
- vertical tangents (∞ gradient)

Tangents & gradients

Tangent line - defined by $y = mx + c$ where $m = \frac{dy}{dx}$
Normal line - $\perp$ tangent ($m_{tan} \cdot m_{\text{norm}} = -1$)
Secant $= \frac{f(x+h)-f(x)}{h}$

Strictly increasing/decreasing

For x_2 and x_1 where $x_2 > x_1$:

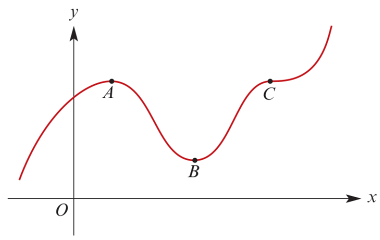
- **strictly increasing** where $f(x_2) > f(x_1)$ or $f'(x) > 0$
- **strictly decreasing** where $f(x_2) < f(x_1)$ or $f'(x) < 0$
- Endpoints are included, even where gradient $= 0$

Solving on CAS

In main : type function. Interactive → Calculation → Line → (Normal | Tan line)
In graph : define function. Analysis → Sketch → (Normal | Tan line). Type x value to solve for a point. Return to show equation for line.

Stationary points

Stationary where $m = 0$.
Find derivative, solve for $\frac{dy}{dx} = 0$



Local maximum at point A

- $f'(x) > 0$ left of A
- $f'(x) < 0$ right of A

Local minimum at point B

- $f'(x) < 0$ left of B
- $f'(x) > 0$ right of B

Stationary point of inflection at C

Function derivatives

$f(x)$	$f'(x)$
kx^n	knx^{n-1}
$g(x) \pm h(x)$	$g'(x) \pm h'(x)$
c	0
$\frac{u}{v}$	$(v\frac{du}{dx} - u\frac{dv}{dx}) \div v^2$
uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
$f \circ g$	$\frac{dy}{du} \cdot \frac{du}{dx}$
$\sin ax$	$a \cos ax$
$\sin(f(x))$	$f'(x) \cdot \cos(f(x))$
$\cos ax$	$-a \sin ax$
$\cos(f(x))$	$f'(x)(-\sin(f(x)))$
e^{ax}	ae^{ax}
$\log_e ax$	$\frac{1}{x}$
$\log_e f(x)$	$\frac{f'(x)}{f(x)}$