Exponentials & Logarithms

Index laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\sqrt[n]{x} = x^{1/n}$$

Logarithm laws

$$\log_a(mn) = \log_a m + \log_a n$$

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a$$

$$\log_a(m^p) = p\log_a m$$

$$\log_a(m^{-1}) = -\log_a m$$

$$\log_a 1 = 0 \text{ and } \log_a a = 1$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

Inverse functions

For $f : \mathbb{R} \to \mathbb{R}, f(x) = a^x$, inverse is:

$$f^{-1} : \mathbb{R}^+ \to \mathbb{R}, f^{-1} = \log_a x$$

Exponentials

e^x natural exponential function

$$e = \lim_{n\to\infty}(1 + \frac{1}{n})^n$$

Modelling

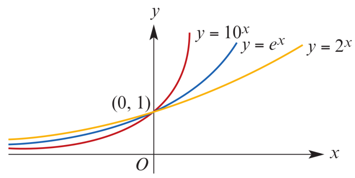
$$A = A_0 e^{kt}$$

- A_0 is initial value
- t is time taken
- k is a constant
- For continuous growth, $k > 0$
- For continuous decay, $k < 0$

Graphing exponential functions

$$f(x) = Aa^{k(x-b)} + c, \quad | \, a > 1$$

- **y-intercept** at $(0, A \cdot a^{-kb} + c)$ as $x \to \infty$
- **horizontal asymptote** at $y = c$
- **domain** is $\mathbb{R}$
- **range** is (c, ∞)
- dilation of factor $|A|$ from x-axis
- dilation of factor $\frac{1}{k}$ from y-axis



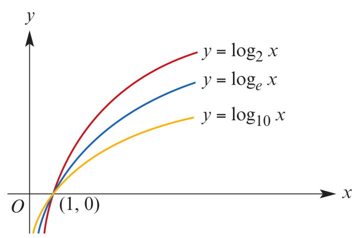
Graphing logarithmic functions

$\log_e x$ is the inverse of e^x (reflection across $y = x$)

$$f(x) = A\log_a k(x - b) + c$$

where

- **domain** is (b, ∞)
- **range** is $\mathbb{R}$
- **vertical asymptote** at $x = b$
- y-intercept exists if $b < 0$
- dilation of factor $|A|$ from x-axis
- dilation of factor $\frac{1}{k}$ from y-axis



Finding equations

On CAS: $\begin{cases} f(3)=9 \\ g(3)=0 \end{cases}\Big|_{a,b}$