Inverse functions

Functions

- vertical line test
- each x value produces only one y value

One to one functions

- $f(x)$ is *one to one* if $f(a) \neq f(b)$ if $a, b \in \text{dom}(f)$ and $a \neq b$
  $\implies$ unique y for each x ($\sin x$ is not 1:1, x^3 is)
- horizontal line test
- if not one to one, it is many to one

Deriving f^{-1}

- if $f(g(x)) = x$, then g is the inverse of f
- reflection across $y - x$
- ran $f =$ dom f^{-1}, dom $f =$ ran f^{-1}
- inverse $\neq$ inverse *function* (i.e. inverse must pass vertical line test)
  $\implies f^{-1}(x)$ exists $\iff f(x)$ is one to one
- $f^{-1}(x) = f(x)$ intersections may lie on line $y = x$

Requirements for showing working for f^{-1}

1. start with *"let $y = f(x)$"*
2. must state *"take inverse"* for line where y and x are swapped
3. do all working in terms of $y = ...$
4. for square root, state $\pm$ solutions then show restricted
5. for inverse *function*, state in function notation