Circular functions

Period of $a\sin(bx)$ is $\frac{2\pi}{b}$

Period of $a\tan(nx)$ is $\frac{\pi}{n}$
Asymptotes at $x = \frac{2k+1)\pi}{2n} \mid k \in \mathbb{Z}$

Reciprocal functions

Cosecant

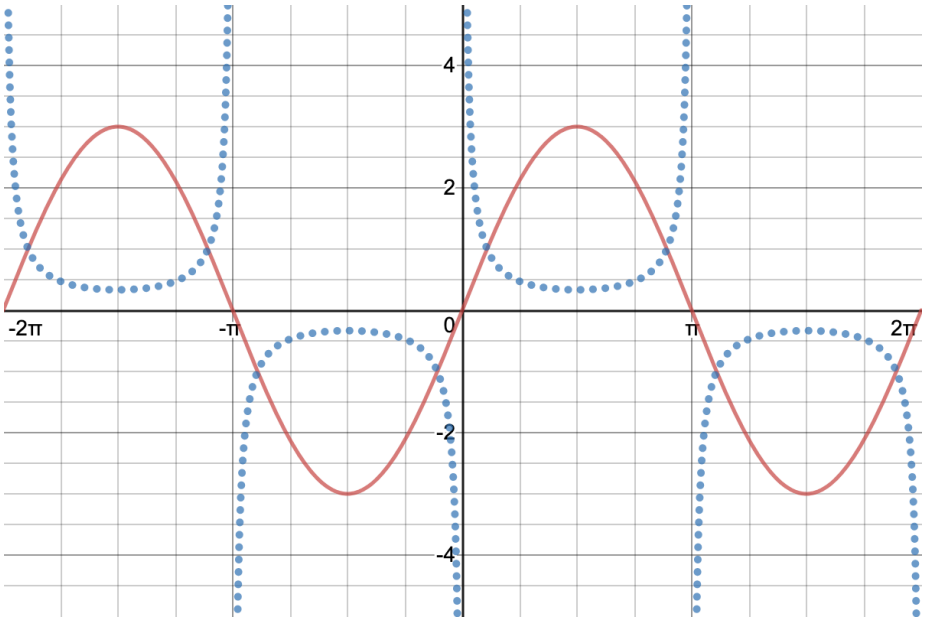

Figure 1:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \mid \sin\theta \neq 0$$

- **Domain** $= \mathbb{R} \quad n\pi : n \in \mathbb{Z}$
- **Range** $= \mathbb{R} \quad (-1, 1)$
- **Turning points** at $\theta = \frac{(2n+1)\pi}{2} \mid n \in \mathbb{Z}$
- **Asymptotes** at $\theta = n\pi \mid n \in \mathbb{Z}$

Secant

!()[graphics/sec.png]

$$\sec\theta = \frac{1}{\cos\theta} \mid \cos\theta \neq 0$$

- **Domain** $= \mathbb{R} \quad \{\frac{(2n+1)\pi}{2} : n \in \mathbb{Z}\}$
- **Range** $= \mathbb{R} \quad (-1, 1)$
- **Turning points** at $\theta = n\pi \mid n \in \mathbb{Z}$
- **Asymptotes** at $\theta = \frac{(2n+1)\pi}{2} \mid n \in \mathbb{Z}$

Cotangent

!()[graphics/cot.png]

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \mid \sin\theta \neq 0$$

- **Domain** $= \mathbb{R} \quad \{n\pi : n \in \mathbb{Z}\}$
- **Range** $= \mathbb{R}$
- **Asymptotes** at $\theta = n\pi \mid n \in \mathbb{Z}$

Symmetry properties

$$\begin{aligned} \sec(\pi \pm x) &= -\sec x \\ \sec(-x) &= \sec x \\ \operatorname{cosec}(\pi \pm x) &= \mp \operatorname{cosec} x \\ \operatorname{cosec}(-x) &= -\operatorname{cosec} x \\ \cot(\pi \pm x) &= \pm \cot x \\ \cot(-x) &= -\cot x \end{aligned} \tag{1}$$

Complementary properties

$$\begin{aligned} \sec\left(\frac{\pi}{2} - x\right) &= \operatorname{cosec} x \\ \operatorname{cosec}\left(\frac{\pi}{2} - x\right) &= \sec x \\ \cot\left(\frac{\pi}{2} - x\right) &= \tan x \\ \tan\left(\frac{\pi}{2} - x\right) &= \cot x \end{aligned} \tag{2}$$

Pythagorean identities

$$\begin{aligned} 1 + \cot^2 x &= \operatorname{cosec}^2 x, \quad \text{where } \sin x \neq 0 \\ 1 + \tan^2 x &= \sec^2 x, \quad \text{where } \cos x \neq 0 \end{aligned} \tag{3}$$

Compound angle formulas

$$\cos(x \pm y) = \cos x + \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Double angle formulas

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned} \tag{4}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$