Transformation

Order of operations: DRT - Dilations, Reflections, Translations

Transforming x^n to $a(x-h)^n + K$

- $|a|$ is the dilation factor of $|a|$ units parallel to y-axis or from x-axis
- if $a < 0$, graph is reflected over x-axis
- k - translation of k units parallel to y-axis or from x-axis
- h - translation of h units parallel to x-axis or from y-axis
- for $(ax)^n$, dilation factor is $\frac{1}{a}$ parallel to x-axis or from y-axis
- when $0 < |a| < 1$, graph becomes closer to axis

Translations

For $y = f(x)$, these processes are equivalent:

- applying the translation $(x, y) \to (x + h, y + k)$ to the graph of $y = f(x)$
- replacing x with $x - h$ and y with $y - k$ to obtain $y - k = f(x - h)$

Dilations

For the graph of $y = f(x)$, there are two pairs of equivalent processes:

1. - Dilating from x-axis: $(x, y) \to (x, by)$
   - Replacing y with $\frac{y}{b}$ to obtain $y = bf(x)$
2. - Dilating from y-axis: $(x, y) \to (ax, y)$
   - Replacing x with $\frac{x}{a}$ to obtain $y = f(\frac{x}{a})$

For graph of $y = \frac{1}{x}$, horizontal & vertical dilations are equivalent (symmetrical).
If $y = \frac{a}{x}$, graph is contracted rather than dilated.

Transforming $f(x)$ to $y = Af[n(x + c)] + b$

Applies to exponential, log, trig, power, polynomial functions.
Functions must be written in form $y = Af[n(x + c)] + b$

A - dilation by factor A from x-axis (if $A < 0$, reflection across y-axis)
n - dilation by factor $\frac{1}{n}$ from y-axis (if $n < 0$, reflection across x-axis)
c - translation from y-axis (x-shift)
b - translation from x-axis (y-shift)

Power functions

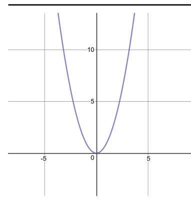
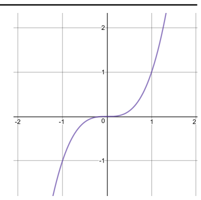
Strictly increasing: $f(x_2) > f(x_1)$ where $x_2 > x_1$ (including $x = 0$)

Odd and even functions

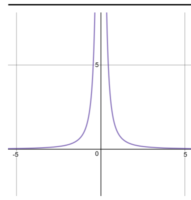
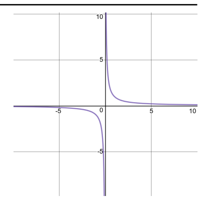
Even when $f(x) = -f(x)$
Odd when $-f(x) = f(-x)$

Function is even if it can be reflected across y-axis $\implies f(x) = f(-x)$
Function $x^{\pm \frac{p}{q}}$ is odd if q is odd

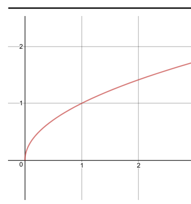
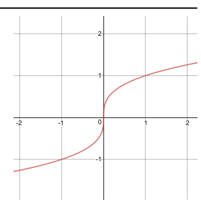
x^n where $n \in \mathbb{Z}^+$

n is even:	n is odd:
	

x^n where $n \in \mathbb{Z}^-$

n is even:	n is odd:
	

$x^{\frac{1}{n}}$ where $n \in \mathbb{Z}^+$

n is even:	n is odd:
	

$x^{\frac{-1}{n}}$ where $n \in \mathbb{Z}^+$

Mostly only on CAS.

We can write $x^{\frac{-1}{n}} = \frac{1}{x^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{x}}$n.

Domain is: $\begin{cases} \mathbb{R} \quad \{0\} & \text{if } n \text{ is odd} \\ \mathbb{R}^+ & \text{if } n \text{ is even} \end{cases}$

If n is odd, it is an odd function.

$x^{\frac{p}{q}}$ where $p, q \in \mathbb{Z}^+$

$$x^{\frac{p}{q}} = \sqrt[q]{x^p}$$

- if $p > q$, the shape of x^p is dominant
- if $p < q$, the shape of $x^{\frac{1}{q}}$ is dominant
- points $(0, 0)$ and $(1, 1)$ will always lie on graph
- Domain is: $\begin{cases} \mathbb{R} & \text{if } q \text{ is odd} \\ \mathbb{R}^+ \cup \{0\} & \text{if } q \text{ is even} \end{cases}$

Combinations of functions (piecewise/hybrid)

$$\text{e.g.} \quad f(x) = \begin{cases} \sqrt[3]{x}, & x \leq 0 \\ 2, & 0 < x < 2 \\ x, & x \geq 2 \end{cases}$$

Open circle - point included
Closed circle - point not included

Sum, difference, product of functions

sum	$f + g$	domain = $\text{dom}(f) \cap \text{dom}(g)$
difference	$f - g$ or $g - f$	domain = $\text{dom}(f) \cap \text{dom}(g)$
product	$f \times g$	domain = $\text{dom}(f) \cap \text{dom}(g)$

Addition of linear piecewise graphs - add y-values at key points

Product functions:

- product will equal 0 if one of the functions is equal to 0
- turning point on one function does not equate to turning point on product

Matrix transformations

Find new point (x', y'). Substitute these into original equation to find image with original variables (x, y).

Composite functions

$(f \circ g)(x)$ is defined iff $\text{ran}(g) \subseteq \text{dom}(f)$