Circular functions

Reciprocal functions

Cosecant

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \mid \sin\theta \neq 0$$

- **Domain** $= \mathbb{R} \setminus n\pi : n \in \mathbb{Z}$
- **Range** $= \mathbb{R} \setminus (-1,1)$
- **Turning points** at $\theta = \frac{(2n+1)\pi}{2} \mid n \in \mathbb{Z}$
- **Asymptotes** at $\theta = n\pi \mid n \in \mathbb{Z}$

Secant

$$\sec\theta = \frac{1}{\cos\theta} \mid \cos\theta \neq 0$$

- **Domain** $= \mathbb{R} \setminus \{\frac{(2n+1)\pi}{2} : n \in \mathbb{Z}\}$
- **Range** $= \mathbb{R} \setminus (-1,1)$
- **Turning points** at $\theta = n\pi \mid n \in \mathbb{Z}$
- **Asymptotes** at $\theta = \frac{(2n+1)\pi}{2} \mid n \in \mathbb{Z}$

Cotangent

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \mid \sin\theta \neq 0$$

- **Domain** $= \mathbb{R} \setminus \{n\pi : n \in \mathbb{Z}\}$
- **Range** $= \mathbb{R}$
- **Asymptotes** at $\theta = n\pi \mid n \in \mathbb{Z}$

Symmetry properties

$$
\begin{aligned}
\sec(\pi \pm x) &= -\sec x \\
\sec(-x) &= \sec x \\
\operatorname{cosec}(\pi \pm x) &= \mp\operatorname{cosec} x \\
\operatorname{cosec}(-x) &= -\operatorname{cosec} x \\
\cot(\pi \pm x) &= \pm\cot x \\
\cot(-x) &= -\cot x
\end{aligned}
\tag{1}
$$

Complementary properties

$$\sec\left(\frac{\pi}{2} - x\right) = \operatorname{cosec} x$$
$$\operatorname{cosec}\left(\frac{\pi}{2} - x\right) = \sec x$$
$$\cot\left(\frac{\pi}{2} - x\right) = \tan x \tag{2}$$
$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Pythagorean identities

$$1 + \cot^2 x = \operatorname{cosec}^2 x, \quad \text{where } \sin x \neq 0$$
$$1 + \tan^2 x = \sec^2 x, \quad \text{where } \cos x \neq 0 \tag{3}$$