Year 12 Methods

Unit 3 Revision Lecture

Monash University
presented by Kevin McMenamin

Andrew Lorimer

5 July 2019

1 Graphs

16 types of graph—put in reference book:

1. truncus
2. hyperbola
3. sqrt
4. parabola
5. cubic
6. quartic
7. linear
8. circle
9. semicircle
10. tan
11. sin
12. cos
13. log
14. exp
15. $x^{\frac{a}{b}}$
16. $x^{\frac{-a}{b}}$

1.1 Power functions

- In first quadrant, shape of graph for $x > 0 \cap y > 0$ is either $\sqrt{x}$ or x^2

1.2 Features of graphs

- Asymptotes
- Intercepts
- Stationary points
- Endpoints
- Other critical points
- Continuous or discontinuous

Key points

- All transformations can be described by matrices
- Inverse is a transformation
- Memorise approximate values of e, π, $\sqrt{2}$, $\sqrt{3}$
- Put 16 base graphs in reference book

2 Transformations

Order: **Reflect** $\longrightarrow$ **Dilate** $\longrightarrow$ **Translate**

2.1 Two forms

- note a and b can be positive or negative
- check validity of solutions for logarithms
- results in transformed equation $y' = f'(x)$

$$y' = a \cdot f(\frac{1}{b}(x' - c)) + d$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

Key points

- All transformations can be described by matrices
- Inverse is a transformation
- Check validity of $\log_a x$ solutions/transformations

3 Calculus

Possible questions:

- Average rate of change
- Instantaneous rate of change
- Tangent line
- Normal line
- Features of gradient function
  - Degree
  - Orientation
  - Format
  - Turning points
  - Inflection points
  - Asymptotes
- Find original function from derivative
  $\longrightarrow$ *Use information to find unknowns*
- Application questions - e.g. Pythagoras, trig. functions, measurement, given eqn

3.1 Integration

3.1.1 Polynomials

$$f(x) = \int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c \,, \quad n \neq -1$$

$$f(x) = \int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c \,, \quad n \neq -1$$

3.1.2 Exponentials

$$f(x) = \int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + c$$

3.1.3 Logarithms

ignore modulus for methods

$$f(x) = \int \frac{1}{x}\,dx = \ln|x| + c$$

$$f(x) = \int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$$

$$f(x) - \int \frac{h'(x)}{h(x)}\,dx = \ln|h(x)| + c \qquad \text{(general form)}$$

3.1.4 Trigonometric functions

$$f(x) = \int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$$

$$f(x) = \int \sin(ax+b)\,dx = -\frac{1}{a}\sin(ax+b) + c$$

$$f(x) = \int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$$

3.2 Area under curves

- **Upper rectangles** (overestimate) vs. **lower rectangles** (underestimate)
- Rotate (invert) graph to make it easier, e.g. $y = \sqrt{x} \longrightarrow x = y^2$

Key points

- For *an* antiderivative, $+c \quad \forall\, c \in \mathbb{R}$ is also acceptable
- Practice multi-part problems e.g:
  a) Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x\sin x$. Find $f'(x)$.
  b) Use the result of (a) to find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\cos x\,dx$ in the form $a\pi + b$.

4 Probability

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \cup B) = 0 \qquad \text{(mutually exclusive)}$$

4.1 Conditional probability

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A \cap B) = \Pr(A|B) \times \Pr(B) \qquad \text{(multiplication theorem)}$$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B) \qquad \text{(independent events)}$$

4.2 Discrete random distributions

Any experiment or activity involving chance will have a probability associated with each result or *outcome*. If the outcomes have a reference to **discrete numeric values** (outcomes that can be counted), and the result is unknown, then the activity is a *discrete random probability distribution.*

4.2.1 Discrete probability distributions

If an activity has outcomes whose probability values are all positive and less than one ($\implies 0 \le p(x) \le 1$), and for which the sum of all outcome probabilities is unity ($\implies \sum p(x) = 1$), then it is called a *probability distribution* or *probability mass* function.

- **Probability distribution graph** - a series of points on a cartesian axis representing results of outcomes. $\Pr(X = x)$ is on y-axis, x is on x axis.
- **Mean** μ - measure of central tendency. *Balance point* or *expected value* of a distribution. Centre of a symmetrical distribution.
- **Variance** σ^2 - measure of spread of data around the mean. Not the same magnitude as the original data. Represented by $\sigma^2 = \text{Var}(x) = \sum(x = \mu)^2 \times p(x) = \sum(x - \mu)^2 \times \Pr(X = x)$. Alternatively: $\sigma^2 = \text{Var}(X) = \sum x^2 \times p(x) - \mu^2$
- **Standard deviation** σ - measure of spread in the original magnitude of the data. Found by taking square root of the variance: $\sigma = \text{sd}(X) = \sqrt{\text{Var}(X)}$

4.3 Binomial distribution (Bernoulli trials)

A type of discrete probability distribution. This distribution has the following characteristics:

1. Samples are taken from a population size that remains constant (*sampling with replacement*)
2. Every result or trial can be classed as either a *success* or *failure*
3. The probability of a succcess is the same from one trial to the next, notated by p
4. The probability of a failure is the complement of the probability of a success, notated by $1 - p$
5. There are a finite number of trials that define the sample size, notated by n

4.3.1 Bernoulli trials

Same properties as above. Number of successes in a finite number of Bernoulli trials is defined as the **binomial distribution**. The distribution can take the form:

$$X \sim \text{Bi}(n, p)$$

Then, the probability values for each value of X follow the rule:

$$p(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$$

4.4 Continuous random distributions

If the outcomes of an activity have a reference to *continuous numeric* values (outcomes that can be measured), then the activity is associated with a **continuous probability distribution**. The probabilities are calculuated by finding the area under the graph between the required x values (integrate).
The probability of a single *outcome value* does not exist for continuous probability distributions.

4.5 Continuous probability distributions

If an experiment or activity has a **function** whose values are all positive ($\implies f(x) \ge 0 \forall x$), and for which the area under the graph between the lowest outcome value and the greatest outcome value is unity ($\implies \int_{\text{lower}}^{\text{upper}} f(x)\, dx = 1$), then it is called a **probability density function**.

Example probability density function: $f(x) = \begin{cases} k(9 - x^2), & 0 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$

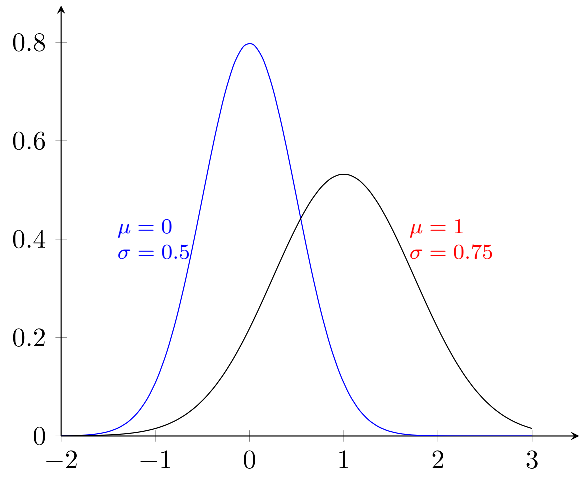

Figure 1: Two *general* normal distributions

4.6 Normal distributions

A very specific and special continuous probability distribution. Characteristics:

- Many sets of data occurring naturally and taken randomly will have a normal distribution
- No single outcome value can be calculated
- Probabilities are found between certain outcome values of the distribution
- The values of the distribution are symmetrical around the mean (μ) and form a bell-shaped curve
- The distribution is best described using its central or mean value, μ, and its measure of spread, σ
- The distribution can take the form $X \sim N(\mu, \sigma^2)$

General normal distribution	Standard normal distribution
$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$