1 Calculus

Average rate of change

$$m \text{ of } x \in [a,b] = \frac{f(b)-f(a)}{b-a} = \frac{dy}{dx}$$

On CAS: Action → Calculation → `diff`

Average value

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Instantaneous rate of change

Secant - line passing through two points on a curve

Chord - line segment joining two points on a curve

Limit theorems

1. For constant function $f(x) = k$, $\lim_{x\to a} f(x) = k$
2. $\lim_{x\to a}(f(x) \pm g(x)) = F \pm G$
3. $\lim_{x\to a}(f(x) \times g(x)) = F \times G$
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{F}{G}, G \neq 0$

A function is continuous if $L^- = L^+ = f(x)$ for all values of x.

First principles derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Not differentiable at:

- discontinuous points
- sharp point/cusp
- vertical tangents (∞ gradient)

Tangents & gradients

Tangent line - defined by $y = mx + c$ where $m = \frac{dy}{dx}$

Normal line - $\perp$ tangent ($m_{tan} \cdot m_{\text{norm}} = -1$)

Secant $= \frac{f(x+h)-f(x)}{h}$

On CAS:

Action → Calculation → Line → `tanLine` or `normal`

Strictly increasing/decreasing

For x_2 and x_1 where $x_2 > x_1$:

- **strictly increasing**

  where $f(x_2) > f(x_1)$ or $f'(x) > 0$
- **strictly decreasing**

  where $f(x_2) < f(x_1)$ or $f'(x) < 0$
- Endpoints are included, even where gradient $= 0$

On CAS

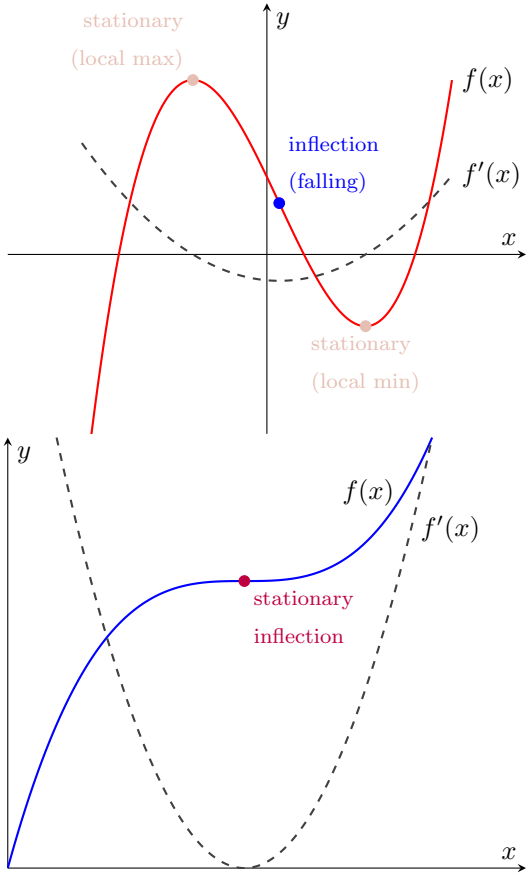
In main: type function. Interactive → Calculation → Line → (Normal | Tan line)

In graph: define function. Analysis → Sketch → (Normal | Tan line). Type x value to solve for a point. Return to show equation for line.

Stationary points

Stationary point: $f'(x) = 0$

Point of inflection: $f'' = 0$



Derivatives

$f(x)$	$f'(x)$	
$\sin x$	$\cos x$	
$\sin ax$	$a\cos ax$	
$\cos x$	$-\sin x$	
$\cos ax$	$-a\sin ax$	
$\tan f(x)$	$f^2(x)\sec^2 f(x)$	
e^x	e^x	
e^{ax}	ae^{ax}	
ax^{nx}	$an\cdot e^{nx}$	
$\log_e x$	$\frac{1}{x}$	
$\log_e ax$	$\frac{1}{x}$	
$\log_e f(x)$	$\frac{f'(x)}{f(x)}$	
$\sin(f(x))$	$f'(x)\cdot\cos(f(x))$	
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	
$\tan^{-1} x$	$\frac{1}{1+x^2}$	
$\frac{d}{dy}f(y)$	$\frac{1}{\frac{dx}{dy}}$	(reciprocal)
uv	$u\frac{dv}{dx}+v\frac{du}{dx}$	(product rule)
$\frac{u}{v}$	$\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	(quotient rule)
$f(g(x))$	$f'(g(x))\cdot g'(x)$	

Antiderivatives

$f(x)$	$\int f(x) \cdot dx$	
k (constant)	$kx + c$	
x^n	$\frac{1}{n+1}x^{n+1}$	
ax^{-n}	$a \cdot \log_e \lvert x \rvert + c$	
$\frac{1}{ax+b}$	$\frac{1}{a}\log_e(ax+b) + c$	
$(ax+b)^n$	$\frac{1}{a(n+1)}(ax+b)^{n-1} + c \mid n \neq 1$	
$(ax+b)^{-1}$	$\frac{1}{a}\log_e \lvert ax+b \rvert + c$	
e^{kx}	$\frac{1}{k}e^{kx} + c$	
e^k	$e^k x + c$	
$\sin kx$	$\frac{-1}{k}\cos(kx) + c$	
$\cos kx$	$\frac{1}{k}\sin(kx) + c$	
$\sec^2 kx$	$\frac{1}{k}\tan(kx) + c$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a} + c \mid a > 0$	
$\frac{-1}{\sqrt{a^2-x^2}}$	$\cos^{-1}\frac{x}{a} + c \mid a > 0$	
$\frac{a}{a^2-x^2}$	$\tan^{-1}\frac{x}{a} + c$	
$\frac{f'(x)}{f(x)}$	$\log_e f(x) + c$	
$\int f(u) \cdot \frac{du}{dx} \cdot dx$	$\int f(u) \cdot du$	(substitution)
$f(x) \cdot g(x)$	$\int [f'(x) \cdot g(x)]dx + \int [g'(x)f(x)]dx$	