Physics Andrew Lorimer

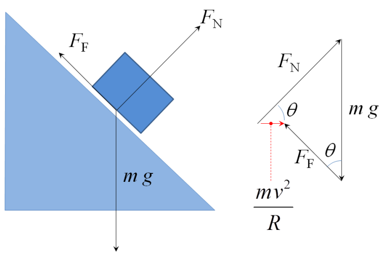
1 Motion

m/s $\times 3.6 =$ km/h

Inclined planes

$F = mg \sin\theta - F_{\text{frict}} = ma$

Banked tracks



$\theta = \tan^{-1} \frac{v^2}{rg}$

ΣF always acts towards centre (horizontally)

$\Sigma F = F_{\text{norm}} + F_{\text{g}} = \frac{mv^2}{r} = mg \tan\theta$

Design speed $v = \sqrt{gr \tan\theta}$

$n \sin\theta = mv^2 \div r, \quad n \cos\theta = mg$

Work and energy

$W = Fs = Fs \cos\theta = \Delta \Sigma E$

$E_K = \frac{1}{2}mv^2$ (kinetic)

$E_G = mgh$ (potential)

$\Sigma E = \frac{1}{2}mv^2 + mgh$ (energy transfer)

Horizontal circular motion

$v = \frac{2\pi r}{T}$

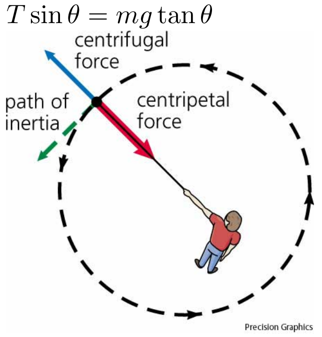
$f = \frac{1}{T}, \quad T = \frac{1}{f}$

$a_{centrip} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$\Sigma F, a$ towards centre, v tangential

$\Sigma F = F_{centrip} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2} =$

$T \sin\theta = mg \tan\theta$



Vertical circular motion

$T + mg = \frac{mv^2}{r}, v = \sqrt{rg}$ (top)

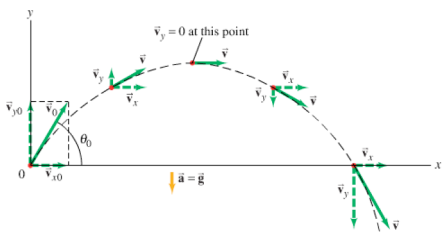
$T - mg = \frac{mv^2}{r}, v = \sqrt{2rg}$ (bottom)

$E_{K\,\text{bottom}} = E_{K\,\text{top}} + mgh$

Projectile motion

- v_x is constant: $v_x = \frac{s}{t}$
- use suvat to find t from y-component
- vertical component gravity: $a_y = -g$

$v = \sqrt{v_x^2 + v_y^2}$	vectors
$h = \frac{u^2 \sin\theta^2}{2g}$	max height
$x = ut\cos\theta$	Δx at t
$y = ut\sin\theta - \frac{1}{2}gt^2$	height at t
$t = \frac{2u\sin\theta}{g}$	time of flight
$d = \frac{v^2}{g}\sin\theta$	horiz. range



Pulley-mass system

$a = \frac{m_2 g}{m_1 + m_2}$ where m_2 is suspended

$\Sigma F = m_2 g - m_1 g = \Sigma ma$ (solve)

Graphs

- Force-time: $A = \Delta\rho$
- Force-disp: $A = W$
- Force-ext: $m = k, \quad A = E_{spr}$
- F_g-dist: $A = \Delta$ gpe
- Field-dist: $A = \Delta$ gpe / kg

Hooke's law

$F = -kx$ (intercepts origin)

elastic potential energy $= \frac{1}{2}kx^2$

$x = \frac{2mg}{k}$

Vertical: $\Delta E = \frac{1}{2}kx^2 + mgh$

Motion equations

	no
$v = u + at$	x
$x = \frac{1}{2}(v+u)t$	a
$x = ut + \frac{1}{2}at^2$	v
$x = vt - \frac{1}{2}at^2$	u
$v^2 = u^2 + 2ax$	t

Momentum

$\rho = mv$

impulse $= \Delta\rho, \quad F\Delta t = m\Delta v$

$\Sigma(mv_0) = (\Sigma m)v_1$ (conservation)

if elastic:

$$\sum_{i=1}^{n} E_K(i) = \sum_{i=1}^{n}(\frac{1}{2}m_i v_{i0}^2) = \frac{1}{2}\sum_{i=1}^{n}(m_i)v_f^2$$

2 Relativity

Postulates

1. Laws of physics are constant in all intertial reference frames
2. Speed of light c is the same to all observers (Michelson-Morley)

$\therefore$ t must dilate as speed changes

high-altitude particles: t dilation means more particles reach Earth than expected (half-life greater when obs. from Earth)

Inertial reference frame $a = 0$

Proper time t_0 | **length** l_0 measured by observer in same frame as events

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

$t = t_0\gamma$ (t longer in moving frame)

$l = \frac{l_0}{\gamma}$ (l contracts $\parallel$ v: shorter in moving frame)

$m = m_0\gamma$ (mass dilation)

Energy and work

Total energy = mass energy

$E_{\text{rest}} = mc^2, \quad E_K = (\gamma - 1)mc^2$

$E_{\text{total}} = \gamma E_{\text{rest}} = E_K + E_{\text{rest}} = \gamma mc^2$

$W = \Delta E = \Delta mc^2 = (\gamma - 1)m_{\text{rest}}c^2$

Relativistic momentum

$$\rho = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mv = \gamma \rho_0$$

$\rho \to \infty$ as $v \to c$

$v = c$ is impossible (requires $E = \infty$)

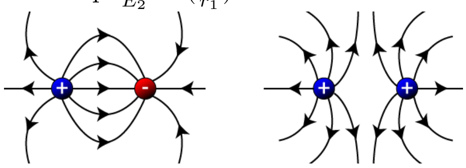
$$v = \frac{\rho}{m\sqrt{1+\frac{p^2}{m^2c^2}}}$$

3 Fields and power

Non-contact forces

- electric (dipoles & monopoles)
- magnetic (dipoles only)
- gravitational (monopoles only, $F_g = 0$ at mid, attractive only)

- monopoles: lines towards centre
- dipoles: field lines $+ \to -$ or N $\to$ S (two magnets) or $\to$ N (single)
- closer field lines means larger force
- dot: out of page, cross: into page
- +ve corresponds to N pole
- Inv. sq. $\frac{E_1}{E_2} = (\frac{r_2}{r_1})^2$



Gravity

$$F_g = G\frac{m_1m_2}{r^2} \quad \text{(grav. force)}$$

$$g = \frac{F_g}{m_2} = G\frac{m_1}{r^2} \quad \text{(field of } m_1\text{)}$$

$$E_g = mg\Delta h \quad \text{(gpe)}$$

$$W = \Delta E_g = Fx \quad \text{(work)}$$

$$w = m(g-a) \quad \text{(app. weight)}$$

Satellites

$$v = \sqrt{\frac{GM}{r}} = \sqrt{gr} = \frac{2\pi r}{T}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = 2\pi\sqrt{\frac{r_{\text{alt}}}{g_{\text{alt}}}} \quad \text{(period)}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \quad \text{(radius)}$$

Magnetic fields

$$F = qvB \quad (F \text{ on moving } q)$$

$$F = IlB \quad (F \text{ of } B \text{ on } I)$$

$$B = \frac{mv}{qr} \quad \text{(field strength on e-)}$$

$$r = \frac{mv}{qB} \quad \text{(radius of } q \text{ in } B\text{)}$$

if $B \not\perp A, \Phi \to 0$, if $B \parallel A, \Phi = 0$

Electric fields

$$F = qE (= ma) \quad \text{(strength)}$$

$$F = k\frac{q_1q_2}{r^2} \quad \text{(force between } q_{1,2}\text{)}$$

$$E = k\frac{q}{r^2} \quad \text{(field on point charge)}$$

$$E = \frac{V}{d} \quad \text{(field between plates)}$$

$$F = BInl \quad \text{(force on a coil)}$$

$$\Phi = B_\perp A \quad \text{(magnetic flux)}$$

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = Blv \quad \text{(induced emf)}$$

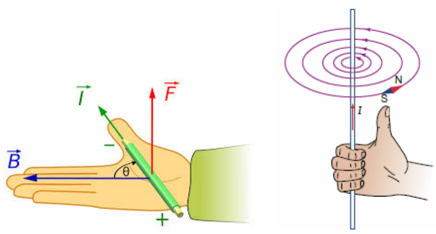
$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \quad \text{(xfmr coil ratios)}$$

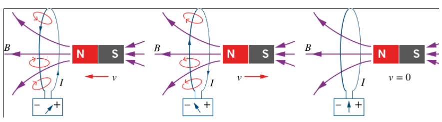
Lenz's law: I_{emf} opposes $\Delta\Phi$ (emf creates I with associated field that opposes $\Delta\Phi$)

Eddy currents: counter movement within a field

Right hand grip: thumb points to I (single wire) or N (solenoid / coil)

Magnet through ring: consider g





Flux-time graphs: $m \times n =$ emf . If f increases, ampl. & f of $\mathcal{E}$ increase

Xfmr core strengthens & focuses Φ

Particle acceleration

$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$

e- accelerated with x V is given x eV

$$W = \frac{1}{2}mv^2 = qV \quad \text{(field or points)}$$

$$V_{\text{point}} = (V_1 - V_2) \div 2 \quad \text{(if midpoint)}$$

$$v = \sqrt{\frac{2qV}{m}} \quad \text{(velocity of particle)}$$

Circular path: $F \perp B \perp v$

Power transmission

$$V_{\text{rms}} = \frac{V_{\text{p}}}{\sqrt{2}} = \frac{V_{\text{p}\to\text{p}}}{2\sqrt{2}}$$

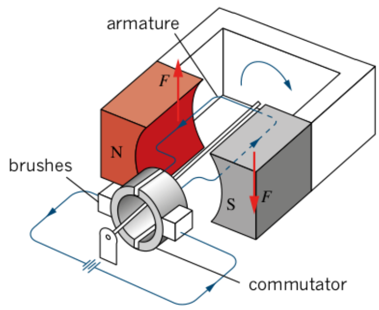
$$P_{\text{loss}} = \Delta VI = I^2R = \frac{\Delta V^2}{R}$$

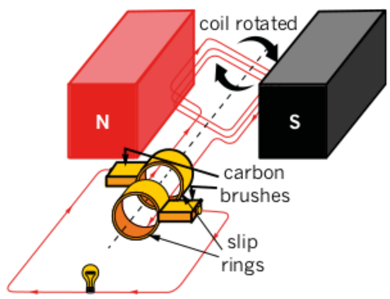
$$V_{\text{loss}} = IR$$

Use high-V side for correct $|V_{drop}|$

- Parallel V is constant
- Series V shared within branch

Motors





Force on I-carying wire, not Cu

$F = 0$ for front back of coil (parallel)

Any angle > 0 will produce force

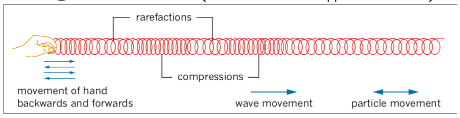
DC: split ring (two halves)

AC: slip ring (separate rings with constant contact)

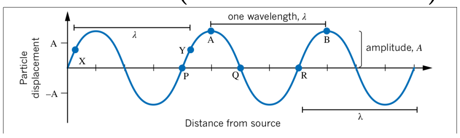
4 Waves

nodes: fixed on graph
amplitude: max disp. from $y = 0$
rarefactions and **compressions**
mechanical: transfer of energy without net transfer of matter

Longitudinal (motion $\parallel$ wave)



Transverse (motion $\perp$ wave)

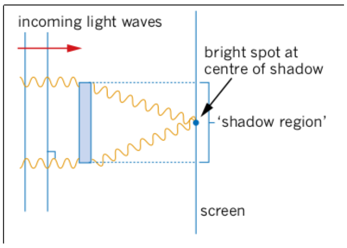


$T = \frac{1}{f}$ (period: time for one cycle)
$v = f\lambda$ (speed: displacement / sec)
$f = \frac{c}{\lambda}$ (for $v = c$)

Doppler effect

When P_1 approaches P_2, each wave w_n has slightly less distance to travel than w_{n-1}. w_n reaches observer sooner than w_{n-1} ("apparent" λ).

Interference

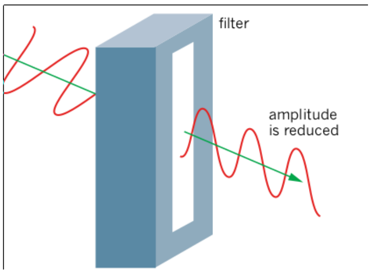


Poissons's spot supports wave theory (circular diffraction)
Standing waves - constructive int. at resonant freq. Rebound from ends.
Coherent - identical frequency, phase, direction (ie strong directional). e.g. laser
Incoherent - e.g. incandescent/LED

Harmonics

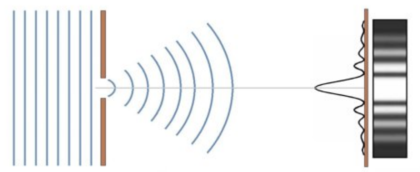
1st harmonic = fundamental
for nodes at both ends:
$\lambda = 2l \div n \qquad f = nv \div 2l$
for node at one end (n is odd):
$\lambda = 4l \div n \qquad f = nv \div 4l$
alternatively, $\lambda = \frac{4l}{2n-1}$ where $n \in \mathbb{Z}$ and $n{+}1$ is the next possible harmonic

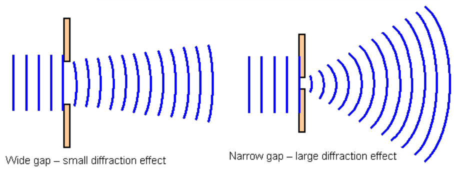
Polarisation



Transverse only. Reduces total A.

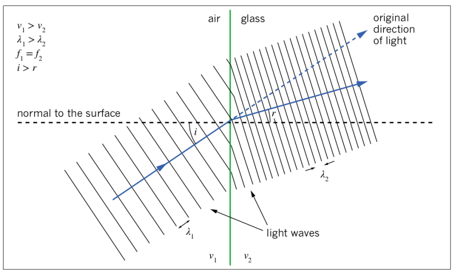
Diffraction





- Constructive: $pd = n\lambda, n \in \mathbb{Z}$
- Destructive: $pd = (n - \frac{1}{2})\lambda, n \in \mathbb{Z}$
- Path difference: $\Delta x = \frac{\lambda l}{d}$ where l = distance from source to observer d = separation between each wave source (e.g. slit) = $S_1 - S_2$
- diffraction $\propto \frac{\lambda}{d} \propto$ fringe spacing
- $d(|\overrightarrow{S_1W}| - |\overrightarrow{S_2W}|) = d\Delta x = \lambda l$
- significant diffraction when $\frac{\lambda}{\Delta x} \geq 1$
- diffraction creates distortion (electron optical microscopes)

Refraction



When a medium changes character, light is *reflected*, *absorbed*, and *transmitted*. λ changes, not f. n changes slightly with f (dispersion)
angle of incidence θ_i = angle of reflection θ_r
Critical angle $\theta_c = \sin^{-1}\frac{n_2}{n_1}$
Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$
$v_1 \div v_2 = \sin\theta_1 \div \sin\theta_2$
$n_1 v_1 = n_2 v_2$
$n = \frac{c}{v}$

5 Light and Matter

Planck's equation

$$E = hf = \frac{hc}{\lambda} = \rho c = qV$$

$h = 6.63 \times 10^{-34}$ Js $= 4.14 \times 10^{-15}$ eVs

$$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$$

De Broglie's theory

$$\lambda = \frac{h}{\rho} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2W}{m}}}$$

$$\rho = \frac{hf}{c} = \frac{h}{\lambda} = mv, \quad E = \rho c$$

$$v = \sqrt{2E_K \div m}$$

- cannot confirm with double-slit (slit $< r_{\text{proton}}$)
- confirmed by e- and x-ray patterns

Force of electrons

$$F = \frac{2P_{\text{in}}}{c}$$

$$\text{photons / sec} = \frac{\text{total energy}}{\text{energy / photon}}$$

$$= \frac{P_{\text{in}}\lambda}{hc} = \frac{P_{\text{in}}}{hf}$$

X-ray electron interaction

- e- stable if $mvr = n\frac{h}{2\pi}$ where $n \in \mathbb{Z}$ and r is radius of orbit
- $\therefore 2\pi r = n\frac{h}{mv} = n\lambda$ (circumference)
- if $2\pi r \neq n\frac{h}{mv}$, no standing wave
- if e- = x-ray diff patterns, $E_{\text{e-}} = \frac{\rho^2}{2m} = (\frac{h}{\lambda})^2 \div 2m$

Photoelectric effect

- V_{supply} does not affect photocurrent
- $V_{\text{sup}} > 0$: attracted to +ve
- $V_{\text{sup}} < 0$: attracted to -ve, $I \to 0$
- v of e- depends on shell
- max I (not V) depends on intensity

Threshold frequency f_0

min f for photoelectron release. if $f < f_0$, no photoelectrons.

Work function $\phi = hf_0$

min E for photoelectron release. determined by strength of bonding. Units: eV or J.

Kinetic energy $E_K = hf - \phi = qV_0$

$V_0 = E_K$ in eV
dashed line below $E_K = 0$

Stopping potential V_0 for min I

$$V_0 = h_{\text{eV}}(f - f_0)$$

Opposes induced photocurrent

Graph features

	m	x-int	y-int
$f \cdot E_K$	h	f_0	$-\phi$
$V \cdot I$		V_0	intensity
$f \cdot V$	$\frac{h}{q}$	f_0	$\frac{-\phi}{q}$

Spectral analysis

- $\Delta E = hf = \frac{hc}{\lambda}$ between ground / excited state
- E and f of photon: $E_2 - E_1 = hf = \frac{hc}{\lambda}$
- Ionisation energy - min E required to remove e-
- EMR is absorbed/emitted when $E_{\text{K-in}} = \Delta E_{\text{shells}}$ (i.e. $\lambda = \frac{hc}{\Delta E_{\text{shells}}}$)
- No. of lines - include all possible states. $\Delta E \neq |\Delta E|$

Uncertainty principle

$\Delta x \approx \frac{\text{slit width}}{2}$
measurement: ρ transferred to e-
slit: possibility of diff. before slit

Wave-particle duality

wave model

- any f works, given t
- predicts delay between incidence and ejection
- speed depends on medium
- supported by bright spot in centre
- $\lambda = \frac{hc}{E}$

particle model

- rate of photoelectron release $\propto$ intensity
- no time delay - one photon releases one electron
- threshold frequency
- double slit: photons interact. interference pattern still appears when a dim light source is used so that only one photon can pass at a time
- light exerts force
- light bent by gravity
- quantised energy
- $\lambda = \frac{h}{\rho}$

6 Experimental design

Absolute uncertainty Δ

(same units as quantity)

$$\Delta(m) = \frac{\mathcal{E}(m)}{100} \cdot m$$

$$(A \pm \Delta A) + (B \pm \Delta A) = (A + B) \pm (\Delta A + \Delta B)$$

$$(A \pm \Delta A) - (B \pm \Delta A) = (A - B) \pm (\Delta A + \Delta B)$$

$$c(A \pm \Delta A) = cA \pm c\Delta A$$

Relative uncertainty $\mathcal{E}$ (unitless)

$$\mathcal{E}(m) = \frac{\Delta(m)}{m} \cdot 100$$

$$(A \pm \mathcal{E}A) \cdot (B \pm \mathcal{E}B) = (A \cdot B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A) \div (B \pm \mathcal{E}B) = (A \div B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A)^n = (A^n \pm n\mathcal{E}A)$$

$$c(A \pm \mathcal{E}A) = cA \pm \mathcal{E}A$$

Uncertainty of a measurement is $\frac{1}{2}$ the smallest division

Precision - concordance of values
Accuracy - closeness to actual value
Random errors - unpredictable, reduced by more tests
Systematic errors - not reduced by more tests
Uncertainty - margin of potential error
Error - actual difference
Hypothesis - can be tested experimentally
Model - evidence-based but indirect representation