Spec - Calculus

Gradients

$$m \text{ of } x \in [a,b] = \frac{f(b)-f(a)}{b-a} = \frac{dy}{dx}$$

Limit theorems

1. For constant function $f(x) = k$, $\lim_{x\to a} f(x) = k$
2. $\lim_{x\to a}(f(x) \pm g(x)) = F \pm G$
3. $\lim_{x\to a}(f(x) \times g(x)) = F \times G$
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{F}{G}, G \neq 0$

First principles derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Tangents & gradients

Tangent line - defined by $y = mx + c$ where $m = \frac{dy}{dx}$
Normal line - $\perp$ tangent ($m_{\text{tan}} \cdot m_{\text{norm}} = -1$)
Secant $= \frac{f(x+h)-f(x)}{h}$

Derivatives

$f(x)$	$f'(x)$
kx^n	knx^{n-1}
$g(x) \pm h(x)$	$g'(x) \pm h'(x)$
c	0
$\frac{u}{v}$	$(v\frac{du}{dx} - u\frac{dv}{dx}) \div v^2$
uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
$f \circ g$	$\frac{dy}{du} \cdot \frac{du}{dx}$
$\sin ax$	$a \cos ax$
$\sin(f(x))$	$f'(x) \cdot \cos(f(x))$
$\cos ax$	$-a \sin ax$
$\cos(f(x))$	$f'(x)(-\sin(f(x)))$
e^{ax}	ae^{ax}
$\log_e ax$	$\frac{1}{x}$
$\log_e f(x)$	$\frac{f'(x)}{f(x)}$

Product rule for $y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Chain rule for $(f \circ g)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$(f \circ g)' = (f' \circ g) \cdot g' = f'(g(x)) \cdot g'(x)$$

Logarithms

$$\log_b(x) = n \quad \text{where} \quad b^n = x$$

Logarithmic identities

$\log_b(xy) = \log_b x + \log_b y$
$\log_b x^n = n \log_b x$
$\log_b y^{x^n} = x^n \log_b y$

Integration

$$\int f(x)dx = F(x) + c$$

area enclosed by curves

$f(x)$	$\int f(x) \cdot dx$
k (constant)	$kx + c$
x^n	$\frac{1}{n+1}x^{n+1} + c$
ax^{-n}	$a \cdot \log_e x + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
e^k	$e^k x + c$
$\sin kx$	$-\frac{1}{k}\cos(kx) + c$
$\cos kx$	$\frac{1}{k}\sin(kx) + c$
$\frac{f'(x)}{f(x)}$	$\log_e f(x) + c$
$g'(x) \cdot f'(g(x)$	$f(g(x))$ (chain rule)
$f(x) \cdot g(x)$	$\int[f'(x) \cdot g(x)]dx + \int[g'(x)f(x)]dx$
$\frac{1}{ax+b}$	$\frac{1}{a}\log_e(ax+b) + c$
$(ax+b)^n$	$\frac{1}{a(n+1)}(ax+b)^{n-1} + c$

Definite integrals

$$\int_a^b f(x) \cdot dx = [F(x)]_a^b = F(b) - F(a)$$

Kinematics

position x - distance from origin or fixed point
displacement s - change in x from starting point
velocity v - change in position with respect to time
acceleration a - change in velocity
speed - magnitude of velocity

	no
$v = u + at$	s
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t
$s = \frac{1}{2}(u+v)t$	a