Exponential and Index Functions

Index laws

$$
\begin{aligned}
a^m \times a^n &= a^{m+n} \\
a^m \div a^n &= a^{m-n}4 \\
(a^m)^n &= a^{m^n} \\
(ab)^m &= a^m b^m \\
(\frac{a}{b})^m &= \frac{a^m}{b^m}
\end{aligned} \tag{1}
$$

Fractional indices

$$\sqrt[n]{x} = x^{1/n}$$

Logarithms

$$\log_b(x) = n \quad \text{where } \ b^n = x$$

Using logs to solve index eq's

Used for equations without common base exponent

Or change base:

$$\log_b c = \frac{\log_a c}{\log_a b}$$

If $a < 1, \quad \log_b a < 0$ (flip inequality operator)

Exponential functions

e^x - natural exponential function

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

Logarithm laws

$$
\begin{aligned}
\log_a(mn) &= \log_a m + \log_a n \\
\log_a(\frac{m}{n}) &= \log_a m - \log_a \\
\log_a(m^p) &= p \log_a m \\
\log_a(m^{-1}) &= -\log_a m \\
\log_a 1 = 0 &\text{ and } \log_a a = 1
\end{aligned} \tag{2}
$$

Inverse functions

For $f : \mathbb{R} \to \mathbb{R}, f(x) = a^x$, inverse is:

$$f^{-1} : \mathbb{R}^+ \to \mathbb{R}, f^{-1} = log_a x$$

Euler's number

$$e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$$

Literal equations

Literal equation - no numerical solutions

Exponential and logarithmic modelling

$$A = A_0 e^{kt}$$

where
A_0 is initial value
t is time taken
k is a constant
For continuous growth, $k > 0$
For continuous decay, $k < 0$

Graphing exponential functions

$$f(x) = Aa^{k(x-b)} + c, \quad | \, a > 1$$

- **y-intercept** at $(0, A \cdot a^{-kb} + c)$ as $x \to \infty$
- **horizontal asymptote** at $y = c$
- **domain** is $\mathbb{R}$
- **range** is (c, ∞)
- dilation of factor A from x-axis
- dilation of factor $\frac{1}{k}$ from y-axis

Graphing logarithmic functions

$log_e x$ is the inverse of e^x (reflection across $y = x$)

$$f(x) = A \log_a k(x - b) + c$$

where

- **domain** is (b, ∞)
- **range** is $\mathbb{R}$
- **vertical asymptote** at $x = b$
- y-intercept exists if $b < 0$
- dilation of factor A from x-axis
- dilation of factor $\frac{1}{k}$ from y-axis