

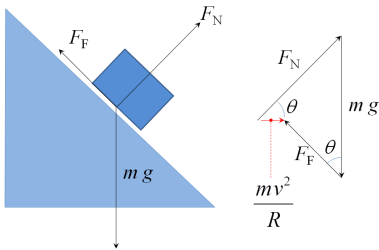
# 1 Motion

$m/s \times 3.6 = km/h$

## Inclined planes

$F = mg \sin \theta - F_{frict} = ma$

## Banked tracks



$\theta = \tan^{-1} \frac{v^2}{rg}$

$\Sigma F$  always acts towards centre (horizontally)

$\Sigma F = F_{norm} + F_g = \frac{mv^2}{r} = mg \tan \theta$

Design speed  $v = \sqrt{gr \tan \theta}$

$n \sin \theta = mv^2 \div r, \quad n \cos \theta = mg$

## Work and energy

$W = Fx = \Delta \Sigma E$  (work)

$E_K = \frac{1}{2}mv^2$  (kinetic)

$E_G = mgh$  (potential)

$\Sigma E = \frac{1}{2}mv^2 + mgh$  (energy transfer)

## Horizontal circular motion

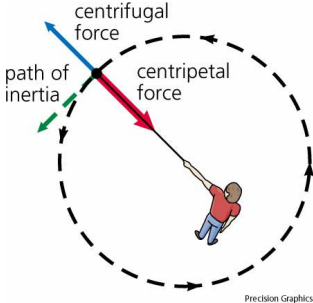
$v = \frac{2\pi r}{T}$

$f = \frac{1}{T}, \quad T = \frac{1}{f}$

$a_{centrip} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$\Sigma F, a$  towards centre,  $v$  tangential

$F_{centrip} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$



## Vertical circular motion

$T$  = tension, e.g. circular pendulum

$T + mg = \frac{mv^2}{r}$  at highest point

$T - mg = \frac{mv^2}{r}$  at lowest point

## Projectile motion

- horizontal component of velocity is constant if no air resistance
- vertical component affected by gravity:  $a_y = -g$

$v = \sqrt{v_x^2 + v_y^2}$  (vectors)

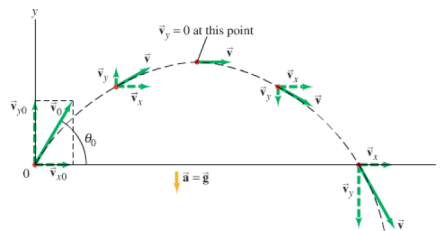
$h = \frac{u^2 \sin^2 \theta}{2g}$  (max height)

$x = ut \cos \theta$  ( $\Delta x$  at  $t$ )

$y = ut \sin \theta - \frac{1}{2}gt^2$  (height at  $t$ )

$t = \frac{2u \sin \theta}{g}$  (time of flight)

$d = \frac{v^2}{g} \sin \theta$  (horiz. range)



## Pulley-mass system

$a = \frac{m_2g}{m_1+m_2}$  where  $m_2$  is suspended

$\Sigma F = m_2g - m_1g = \Sigma ma$  (solve)

## Graphs

- Force-time:  $A = \Delta p$
- Force-disp:  $A = W$
- Force-ext:  $m = k, \quad A = E_{spr}$
- Force-dist:  $A = \Delta gpe$
- Field-dist:  $A = \Delta gpe / kg$

## Hooke's law

$F = -kx$

elastic potential energy =  $\frac{1}{2}kx^2$

$x = \frac{2mg}{k}$

## Motion equations

	no
$v = u + at$	$x$
$x = \frac{1}{2}(v + u)t$	$a$
$x = ut + \frac{1}{2}at^2$	$v$
$x = vt - \frac{1}{2}at^2$	$u$
$v^2 = u^2 + 2ax$	$t$

## Momentum

$\rho = mv$

impulse =  $\Delta \rho, \quad F\Delta t = m\Delta v$

$\Sigma mv_0 = \Sigma mv_1$  (conservation)

$\Sigma E_K \text{ before} = \Sigma E_K \text{ after}$  if elastic

$n$ -body collisions:  $\rho$  of each body is independent

# 2 Relativity

## Postulates

- Laws of physics are constant in all inertial reference frames
  - Speed of light  $c$  is the same to all observers (Michelson-Morley)
- $\therefore t$  must dilate as speed changes

**Inertial reference frame**  $a = 0$

**Proper time**  $t_0$  | **length**  $l_0$  measured by observer in same frame as events

## Lorentz factor

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$t = t_0 \gamma$  ( $t$  longer in moving frame)

$l = \frac{l_0}{\gamma}$  ( $l$  contracts  $\parallel v$ : shorter in moving frame)

$m = m_0 \gamma$  (mass dilation)

$v = c \sqrt{1 - \frac{1}{\gamma^2}}$

## Energy and work

$E_0 = mc^2$  (rest)

$E_{total} = E_K + E_{rest} = \gamma mc^2$

$E_K = (\gamma - 1)mc^2$

$W = \Delta E = \Delta mc^2$

### Relativistic momentum

$$\rho = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mv = \gamma \rho_0$$

$\rho \rightarrow \infty$  as  $v \rightarrow c$

$v = c$  is impossible (requires  $E = \infty$ )

$$v = \frac{\rho}{m\sqrt{1 + \frac{\rho^2}{m^2 c^2}}}$$

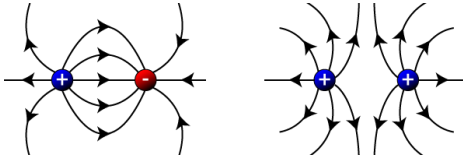
### High-altitude muons

- $t$  dilation more muons reach Earth than expected
- normal half-life  $2.2 \mu s$  in stationary frame,  $> 2.2 \mu s$  observed from Earth

## 3 Fields and power

### Non-contact forces

- electric fields (dipoles & monopoles)
- magnetic fields (dipoles only)
- gravitational fields (monopoles only)
- monopoles: lines towards centre
- dipoles: field lines  $+ \rightarrow -$  or  $N \rightarrow S$  (or perpendicular to wire)
- closer field lines means larger force
- dot: out of page, cross: into page
- +ve corresponds to N pole



### Gravity

$$F_g = G \frac{m_1 m_2}{r^2} \quad (\text{grav. force})$$

$$g = \frac{F_g}{m_2} = G \frac{m_1}{r^2} \quad (\text{field of } m_1)$$

$$E_g = mg\Delta h \quad (\text{gpe})$$

$$W = \Delta E_g = Fx \quad (\text{work})$$

$$w = m(g - a) \quad (\text{app. weight})$$

### Satellites

$$v = \sqrt{\frac{Gm_{\text{planet}}}{r}} = \sqrt{gr} = \frac{2\pi r}{T}$$

$$T = \frac{\sqrt{4\pi^2 r^3}}{GM_{\text{planet}}} \quad (\text{period})$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \quad (\text{radius})$$

### Magnetic fields

- field strength  $B$  measured in tesla
- magnetic flux  $\Phi$  measured in weber
- charge  $q$  measured in coulombs
- emf  $\mathcal{E}$  measured in volts

$$F = qvB \quad (F \text{ on moving } q)$$

$$F = IlB \quad (F \text{ of } B \text{ on } I)$$

$$B = \frac{mv}{qr} \quad (\text{field strength on e-})$$

$$r = \frac{mv}{qB} \quad (\text{radius of } q \text{ in } B)$$

if  $B \perp A, \Phi \rightarrow 0$  , if  $B \parallel A, \Phi = 0$

### Electric fields

$$F = qE \quad (E = \text{strength})$$

$$F = k \frac{q_1 q_2}{r^2} \quad (\text{force between } q_{1,2})$$

$$E = k \frac{q}{r^2} \quad (\text{field on point charge})$$

$$E = \frac{V}{d} \quad (\text{field between plates})$$

$$F = BIl \quad (\text{force on a coil})$$

$$\Phi = B_{\perp} A \quad (\text{magnetic flux})$$

$$\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} \quad (\text{induced emf})$$

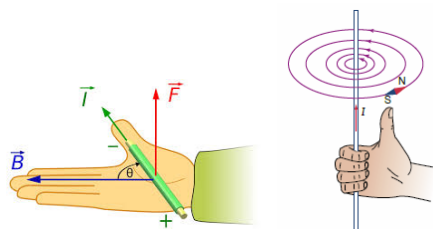
$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \quad (\text{xfmr coil ratios})$$

**Lenz's law:**  $I_{\text{emf}}$  opposes  $\Delta \Phi$

(emf creates  $I$  with associated field that opposes  $\Delta \phi$ )

**Eddy currents:** counter movement within a field

**Right hand grip:** thumb points to  $I$  (single wire) or  $N$  (solenoid / coil)



**Flux-time graphs:**  $m \times n = \text{emf}$

**Transformers:** core strengthens & focuses  $\Phi$

### Particle acceleration

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

e- accelerated with  $x \text{ V}$  is given  $x \text{ eV}$

$$W = \frac{1}{2}mv^2 = qV \quad (\text{field or points})$$

$$v = \sqrt{\frac{2qV}{m}} \quad (\text{velocity of particle})$$

### Power transmission

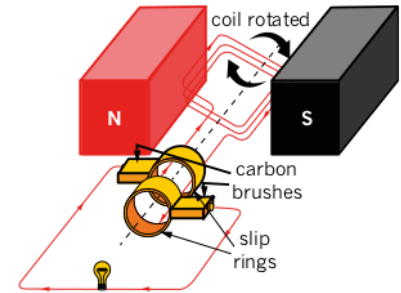
$$V_{\text{rms}} = \frac{V_{\text{p} \rightarrow \text{p}}}{\sqrt{2}}$$

$$P_{\text{loss}} = \Delta VI = I^2 R = \frac{\Delta V^2}{R}$$

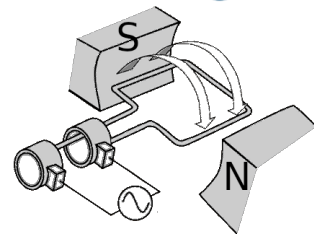
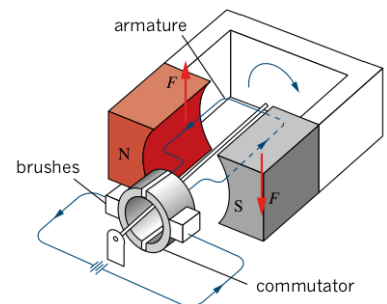
$$V_{\text{loss}} = IR$$

Use high- $V$  side for correct  $|V_{\text{drop}}|$

- Parallel  $V$  is constant
- Series  $V$  shared within branch



### Motors



Force on current-carrying wire, not copper

$F = 0$  for front back of coil (parallel)

Any angle  $> 0$  will produce force

**DC:** split ring (two halves)

**AC:** slip ring (separate rings with constant contact)

## 4 Waves

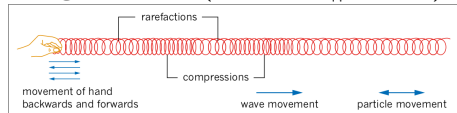
**nodes:** fixed on graph

**amplitude:** max disp. from  $y = 0$

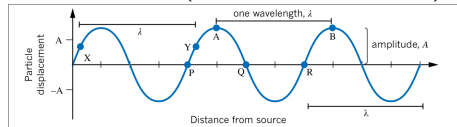
**rarefactions and compressions**

**mechanical:** transfer of energy without net transfer of matter

### Longitudinal (motion || wave)



### Transverse (motion ⊥ wave)



$T = \frac{1}{f}$  (period: time for one cycle)

$v = f\lambda$  (speed: displacement / sec)

## Doppler effect

When  $P_1$  approaches  $P_2$ , each wave  $w_n$  has slightly less distance to travel than  $w_{n-1}$ .  $w_n$  reaches observer sooner than  $w_{n-1}$  ("apparent"  $\lambda$ ).

## Interference

**Standing waves** - constructive int. at resonant freq

**Coherent** - identical frequency, phase, direction (ie strong directional). e.g. laser

**Incoherent** - e.g. incandescent bulb

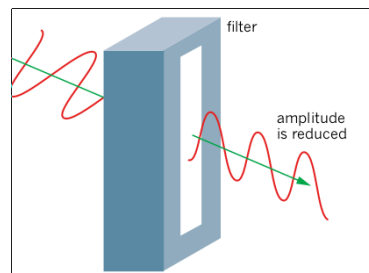
## Harmonics

$\lambda = al \div n$  ( $\lambda$  for  $n^{th}$  harmonic)

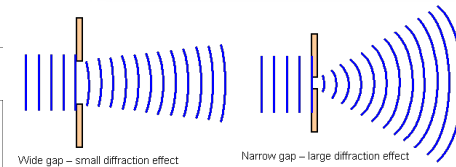
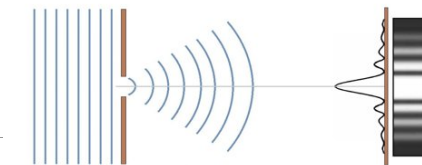
$f = nv \div al$  ( $f$  for  $n^{th}$  harmonic at length  $l$  and speed  $v$ )

where  $a = 2$  for antinodes at both ends,  $a = 4$  for antinodes at one end

## Polarisation

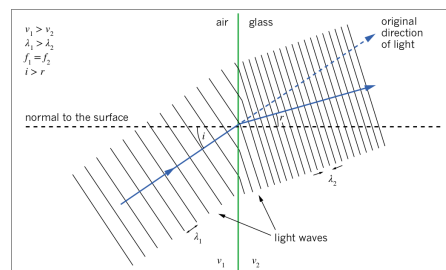


## Diffraction



- Constructive:  $pd = n\lambda, n \in \mathbb{Z}$
- Destructive:  $pd = (n - \frac{1}{2})\lambda, n \in \mathbb{Z}$
- Path difference:  $\Delta x = \frac{\lambda}{d}$  where  $l =$  distance from source to observer  $d =$  separation between each wave source (e.g. slit)  $= S_1 - S_2$
- significant diffraction when  $\frac{\lambda}{\Delta x} \geq 1$
- diffraction creates distortion (electron  $>$  optical microscopes)

## Refraction



When a medium changes character, energy is *reflected*, *absorbed*, and *transmitted*

angle of incidence  $\theta_i =$  angle of reflection  $\theta_r$

Critical angle  $\theta_c = \sin^{-1} \frac{n_2}{n_1}$

Snell's law  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

## 5 Light and Matter

### Planck's equation

$$f = \frac{c}{\lambda}, \quad E = hf = \frac{hc}{\lambda} = \rho c$$

$$h = 6.63 \times 10^{-34} \text{ Js} = 4.14 \times 10^{-15} \text{ eVs}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

### Force of electrons

$$F = \frac{2P_{in}}{c}$$

$$\text{photons / sec} = \frac{\text{total energy}}{\text{energy / photon}}$$

$$= \frac{P_{in}\lambda}{hc} = \frac{P_{in}}{hf}$$

### Photoelectric effect

- $V_{supply}$  does not affect photocurrent
- $V_{sup} > 0$ : e- attracted to collector anode
- $V_{sup} < 0$ : attracted to illuminated cathode,  $I \rightarrow 0$
- $v$  of depends on ionisation energy (shell)
- max current depends on intensity

### Threshold frequency $f_0$

Minimum  $f$  for photoelectrons to be ejected.  $x$ -intercept of frequency vs  $E_K$  graph. if  $f < f_0$ , no photoelectrons are detected.

### Work function $\phi$

Minimum  $E$  required to release photoelectrons. Magnitude of  $y$ -intercept of  $f$  vs  $E_K$  graph.  $\phi$  is determined by strength of bonding. Units: eV or J.

$$\phi = hf_0$$

### Kinetic energy

$$E_{K-max} = hf - \phi$$

voltage in circuit or stopping voltage  $= \max E_K$  in eV

$E_K = x$ -int of  $V \cdot I$  graph (in eV)

dashed line below  $E_K = 0$  ( $E_K < 0$  is impossible)

### Stopping potential $V$ for min $I$

$$V = h_{eV}(f - f_0)$$

## De Broglie's theory

$$\lambda = \frac{h}{\rho} = \frac{h}{mv}$$

$$\rho = \frac{hf}{c} = \frac{h}{\lambda} = mv, \quad E = \rho c$$

- cannot confirm with double-slit (slit  $< r_{\text{proton}}$ )
- confirmed by similar e- and x-ray diff patterns

## X-ray electron interaction

- e- is only stable if  $mvr = n\frac{h}{2\pi}$  where  $n \in \mathbb{Z}$
- rearranging this,  $2\pi r = n\frac{h}{mv} = n\lambda$  (circumference)
- if  $2\pi r \neq n\frac{h}{mv}$ , no standing wave
- if e- = x-ray diff patterns,  $E_{e^-} = \frac{p^2}{2m} = (\frac{h}{\lambda})^2 \div 2m$
- calculating  $h$ :  $\lambda = \frac{h}{\rho}$

## Spectral analysis

- $\Delta E = hf = \frac{hc}{\lambda}$  between ground / excited state
- $E$  and  $f$  of photon:  $E_2 - E_1 = hf = \frac{hc}{\lambda}$
- Ionisation energy - min  $E$  required to remove e-
- EMR is absorbed/emitted when  $E_{K\text{-in}} = \Delta E_{\text{shells}}$  (i.e.  $\lambda = \frac{hc}{\Delta E_{\text{shells}}}$ )
- No. of lines - include all possible states

## Uncertainty principle

measuring location of an e- requires hitting it with a photon, but this causes  $\rho$  to be transferred to electron, moving it.

## Wave-particle duality

### wave model

- cannot explain photoelectric effect
- $f$  is irrelevant to photocurrent
- predicts delay between incidence and ejection
- speed depends on medium

### particle model

- explains photoelectric effect
- rate of photoelectron release  $\propto$  intensity
- no time delay - one photon releases one electron
- double slit: photons interact. interference pattern still appears when a dim light source is used so that only one photon can pass at a time
- light exerts force
- light bent by gravity
- quantised energy

## 6 Experimental design

**Absolute uncertainty  $\Delta$**   
(same units as quantity)

$$\Delta(m) = \frac{\mathcal{E}(m)}{100} \cdot m$$

$$(A \pm \Delta A) + (B \pm \Delta A) = (A+B) \pm (\Delta A + \Delta B)$$

$$(A \pm \Delta A) - (B \pm \Delta A) = (A-B) \pm (\Delta A + \Delta B)$$

$$c(A \pm \Delta A) = cA \pm c\Delta A$$

**Relative uncertainty  $\mathcal{E}$**  (unitless)

$$\mathcal{E}(m) = \frac{\Delta(m)}{m} \cdot 100$$

$$(A \pm \mathcal{E}A) \cdot (B \pm \mathcal{E}B) = (A \cdot B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A) \div (B \pm \mathcal{E}B) = (A \div B) \pm (\mathcal{E}A + \mathcal{E}B)$$

$$(A \pm \mathcal{E}A)^n = (A^n \pm n\mathcal{E}A)$$

$$c(A \pm \mathcal{E}A) = cA \pm \mathcal{E}A$$

Uncertainty of a measurement is  $\frac{1}{2}$  the smallest division

**Precision** - concordance of values

**Accuracy** - closeness to actual value

**Random errors** - unpredictable, reduced by more tests

**Systematic errors** - not reduced by more tests

**Uncertainty** - margin of potential error

**Error** - actual difference

**Hypothesis** - can be tested experimentally

**Model** - evidence-based but indirect representation