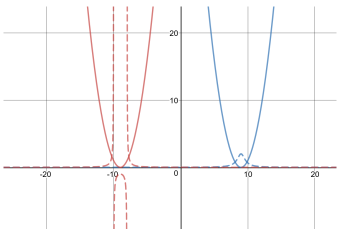
Graphing techniques

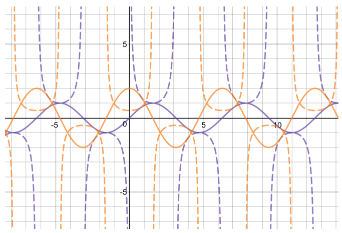
Reciprocal continuous functions

If $y = f(x)$, the reciprocal function is:

$$y = \frac{1}{f(x)}$$

As $f(x) \to \pm\infty$, $\frac{1}{f(x)} \to 0^{\pm}$ (vert asymptote at $f(x) = 0$)





- reciprocal functions are always on the same side of $x = 0$
- if $y = f(x)$ has a local max|min at $x = 1$, then $y = \frac{1}{f(x)}$ has a local max|min at $x = a$
- point of inflection at $P(1, 1)$

Locus of points

- set of points that satisfy a given condition
- path traced by a point that moves according to a condition
- graph on CAS - **conics**

Circular loci

point $P(x, y)$ has a constant distance r from point $C(a, b)$ (centre)

$$PC = r$$

$$(x - a)^2 + (y - b)^2 = r^2$$

Linear loci

$$QP = RP$$

$$\sqrt{(x_Q - q_P)^2 + (y_Q - y_P)^2} = \sqrt{(x_R - x_P)^2 + (y_R - y_P)^2}$$

points Q and R are fixed and have a perpendicular bisector QR. Therefore, any point on line $y = mx + c$ is equidistant from QP and RP.

Since the bisector of the line joining points Q and R is perpendicular to $QR $:

$$m(QR) \times m(RP) = -1$$

Parabolic loci

$$PD = PF$$

$$|y - z| = \sqrt{(x - x_F)^2 + (y - y_F)^2}$$

$$(y - z)^2 = (x - x_F)^2 + (y - y_F)^2$$

Distance of point $P(x, y)$ from fixed point $F(a, b)$ is equal to the distance of P from $y = z \perp$.

Fixed point F is the **focus** (halfway between $y = z$ and $y = y_P$)

Fixed line $x = z$ is the **directrix**

Elliptical loci

Point P moves so that the sum of its distances from two fixed points F_1 and F_2 is a constant k.

$$F_1P + F_2P = k$$

Two foci at F_1 and F_2

Cartesian equation for ellipses:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

centered at (h, k). Width is $2a$, height is $2b$.

Transformations

$$(x, y) \to (x\prime, y\prime)$$

where $x\prime$ and $y\prime$ are the transformation factors (dilation away from x-axis means coefficient of y increases in $y\prime$, and vice versa).

Transformed equation is the same as initial equation with each term divided by its dilation coefficients (must be in terms of $x\prime$ and $y\prime$).

e.g.

$x^2 + y^2 = 1$ is dilated 3 from x, 5 from y. Transformation rule is $(x\prime, y\prime) = (5x, 3y)$ $x = \frac{x\prime}{5}, \quad y = \frac{y\prime}{3}$

Equation $x^2 + y^2 = 1$ becomes

$$\frac{(x\prime)^2}{25} + \frac{(y\prime)^2}{9} = 1$$

Hyperbolic loci

$$|(F_2P - F_1P)| = k$$

Cartesian equation for hyperbolas (a and b are dilation factors):

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Distance between vertices is $2a$ Vertices given by $(h \pm a, k)$

Asymptotes at $y = \pm\frac{b}{a}(x-h) + k$ To make hyperbola up/down rather than left/right, swap x and y

$y^2 - x^2 = 1$ produces hyperbola shifted 90 ° (top and bottom of asymptotes)

Parametric equations

Parametric curve:

$$x = f(t), \quad y = g(t)$$

t is the parameter

To convert to cartesian, solve like simultaneous equations

Polar coordinates

$$x = r\cos\theta, \quad y = r\sin\theta$$

Spirals

$$r = \frac{\theta}{n\pi}$$

- solve intercepts for multiples of $\frac{\pi}{2}$ - or draw table of values for r and θ for each $\frac{n\pi}{2}$

Circles

$$r = a$$

Lines

Horizontal: $r = \frac{n}{\sin\theta}$ Vertical: $r = \frac{n}{\cos\theta}$

Cardioids

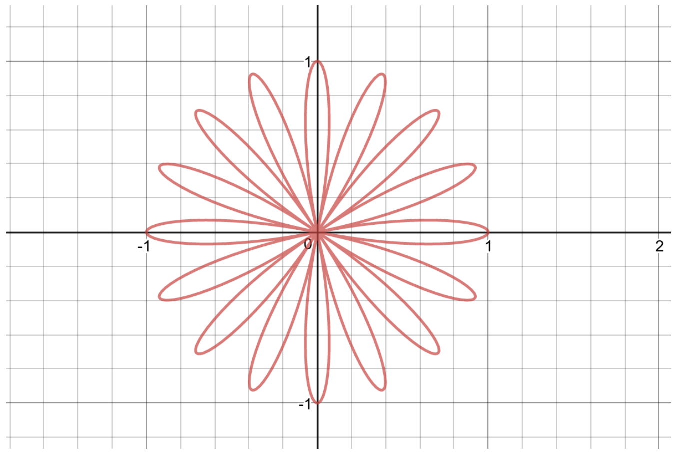
$$r = a(n + \cos\theta)$$

Roses

$$r = \cos(k\theta)$$

If k is odd, half of the petals will overlap (hence there are n petals)

If k is even, petals will not overlap (hence $2n$ petals)



Solving polar graphs

solve in terms of r

e.g. $x = 4$

$r\cos\theta = 4$

$r = \frac{4}{\cos\theta}$

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e.g. $y = x^2$

$r\sin\theta = r^2\cos^2\theta$

$\sin\theta = r\cos^2\theta$

$r = \frac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta$

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e.g. $r = 6\cos\theta$ *(multiply by r)*

$r^2 = 6r\cos\theta$

$x^2 + y^2 = 6x$

complete the square