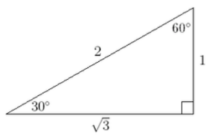
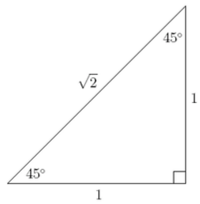
Circular functions

Exact values





sin and cos graphs

$$f(x) = a\sin(bx - c) + d$$
$$f(x) = a\cos(bx - c) + d$$

where

- a is the y-dilation (amplitude)
- b is the x-dilation (period)
- c is the x-shift (phase)
- d is the y-shift (equilibrium position)

Domain is $\mathbb{R}$

Range is $[-b + c, b + c]$;

Graph of $\cos(x)$ starts at $(0, 1)$. Graph of $\sin(x)$ starts at $(0, 0)$.

Mean / equilibrium: line that the graph oscillates around ($y = d$)

Amplitude

Graph oscillates between $+a$ and $-a$ in y-axis

$a = 0$ produces straight line

$a < 0$ inverts the phase (sin becomes cos, vice vera)

Period

Period T is $\frac{2\pi}{b}$

$b = 0$ produces straight line

$b < 0$ inverts the phase

Phase

c moves the graph left-right in the x axis.

If $c = T = \frac{2\pi}{b}$, the graph has no actual phase shift.

Symmetry

$$\sin(\theta + \frac{\pi}{2}) = \sin\theta$$
$$\sin(\theta + \pi) = -\sin\theta$$

$$\cos(\theta + \frac{\pi}{2}) = -\cos\theta$$
$$\cos(\theta + \pi) = -cos(\theta + \frac{3\pi}{2}) = \cos(-\theta)$$

Pythagorean identity

$$\cos^2\theta + \sin^2\theta = 1$$

Complementary relationships

$$\sin(\frac{\pi}{2} - \theta) = \cos\theta$$
$$\cos(\frac{\pi}{2} - \theta) = \sin\theta$$

$$\sin\theta = -\cos(\theta + \frac{\pi}{2})$$
$$\cos\theta = \sin(\theta + \frac{\pi}{2})$$

tan graph

$$y = a\tan(nx)$$

where

- a is x-dilation (period)
- n is y-dilation ($\equiv$ amplitude)
- period T is $\frac{\pi}{n}$
- range is R
- roots at $x = \frac{k\pi}{n}$
- asymptotes at $x = \frac{(2k+1)\pi}{2n}, \quad k \in \mathbb{Z}$

Asymptotes should always have equations and arrow pointing up

Solving trig equations

1. Solve domain for $n\theta$
2. Find solutions for $n\theta$
3. Divide solutions by n

$\sin 2\theta = \frac{\sqrt{3}}{2}, \quad \theta \in [0, 2\pi] \quad (\therefore 2\theta \in [0, 4\pi])$

$2\theta = \sin^{-1}\frac{\sqrt{3}}{2}$

$2\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$

$\therefore \theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$