

# Dynamics

## 1 Resolution of forces

**Resultant force** is sum of force vectors

### In angle-magnitude form

Cosine rule:  $c^2 = a^2 + b^2 - 2ab \cos \theta$       Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

### In $i$ — $j$ form

Vector of  $a$  N at  $\theta$  to  $x$  axis is equal to  $a \cos \theta \mathbf{i} + a \sin \theta \mathbf{j}$ . Convert all force vectors then add.

To find angle of an  $a\mathbf{i} + b\mathbf{j}$  vector, use  $\theta = \tan^{-1} \frac{b}{a}$

### Resolving in a given direction

The resolved part of a force  $P$  at angle  $\theta$  is has magnitude  $P \cos \theta$

To convert force  $\|\vec{OA}$  to angle-magnitude form, find component  $\perp \vec{OA}$  then  $|\mathbf{r}| = \sqrt{(\|\vec{OA}\|)^2 + (\perp \vec{OA})^2}$ ,  $\theta = \tan^{-1} \frac{\perp \vec{OA}}{\|\vec{OA}\|}$

## 2 Newton's laws

1. Velocity is constant without a net external velocity
2.  $\frac{d}{dt}\rho \propto \Sigma F \implies \mathbf{F} = m\mathbf{a}$
3. Equal and opposite forces

### Weight

A mass of  $m$  kg has force of  $mg$  acting on it

### Momentum $\rho$

$$\rho = mv \quad (\text{units kg m/s or Ns})$$

### Reaction force $R$

- With no vertical velocity,  $R = mg$
- With upwards acceleration,  $R - mg = ma$
- With force  $F$  at angle  $\theta$ , then  $R = mg - F \sin \theta$

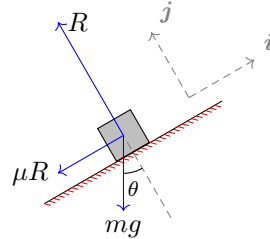
### Friction

$$F_R = \mu R \quad (\text{friction coefficient})$$

### 3 Inclined planes

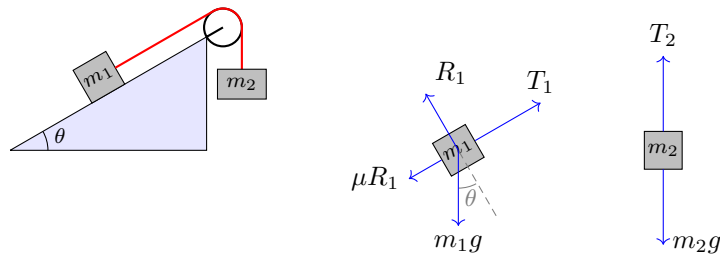
$$\mathbf{F} = |\mathbf{F}| \cos \theta \mathbf{i} + |\mathbf{F}| \sin \theta \mathbf{j}$$

- Normal force  $R$  is at right angles to plane
- Let direction up the plane be  $\mathbf{i}$  and perpendicular to plane  $\mathbf{j}$



### 4 Connected particles

- **Suspended pulley:** tension in both sections of rope are equal
- **Linear connection:** find acceleration of system first
- **Pulley on right angle:**  $a = \frac{m_2 g}{m_1 + m_2}$  where  $m_2$  is suspended (frictionless on both surfaces)
- **Pulley on edge of incline:** find downwards force  $W_2$  and components of mass on plane



### 5 Equilibrium

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \quad \text{(Lami's theorem)}$$

Three methods:

1. Lami's theorem (sine rule)
2. Triangle of forces or CAS (use to verify)
3. Resolution of forces ( $\Sigma F = 0$  - simultaneous)

On CAS: use Geometry, lock known constants.

### 6 Variable forces (DEs)

$$a = \frac{d^2 x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$