Transformations

Order of operations: DRT

dilations — reflections — translations

Transforming x^n to $a(x-h)^n+K$

- dilation factor of $|a|$ units parallel to y-axis or from x-axis
- if $a<0$, graph is reflected over x-axis
- translation of k units parallel to y-axis or from x-axis
- translation of h units parallel to x-axis or from y-axis
- for $(ax)^n$, dilation factor is $\frac{1}{a}$ parallel to x-axis or from y-axis
- when $0<|a|<1$, graph becomes closer to axis

Transforming $f(x)$ to $y=Af[n(x+c)]+b$

Applies to exponential, log, trig, e^x, polynomials.
Functions must be written in form $y=Af[n(x+c)]+b$

- dilation by factor $|A|$ from x-axis (if $A<0$, reflection across y-axis)
- dilation by factor $\frac{1}{n}$ from y-axis (if $n<0$, reflection across x-axis)
- translation of c units from y-axis (x-shift)
- translation of b units from x-axis (y-shift)

Dilations

Two pairs of equivalent processes for $y=f(x)$:

1. - Dilating from x-axis: $(x,y)\to(x,by)$
   - Replacing y with $\frac{y}{b}$ to obtain $y=bf(x)$
2. - Dilating from y-axis: $(x,y)\to(ax,y)$
   - Replacing x with $\frac{x}{a}$ to obtain $y=f(\frac{x}{a})$

For graph of $y=\frac{1}{x}$, horizontal & vertical dilations are equivalent (symmetrical). If $y=\frac{a}{x}$, graph is contracted rather than dilated.

Matrix transformations

Find new point (x',y'). Substitute these into original equation to find image with original variables (x,y).

Reflections

- Reflection **in** axis = reflection **over** axis = reflection **across** axis
- Translations do not change

Translations

For $y=f(x)$, these processes are equivalent:

- applying the translation $(x,y)\to(x+h,y+k)$ to the graph of $y=f(x)$
- replacing x with $x-h$ and y with $y-k$ to obtain $y-k=f(x-h)$

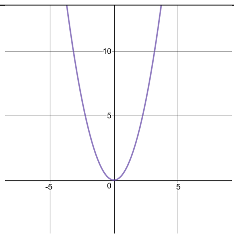
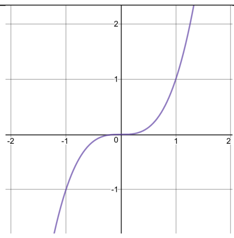
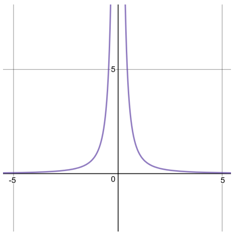
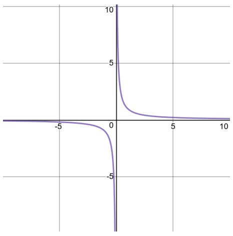
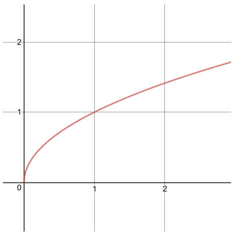
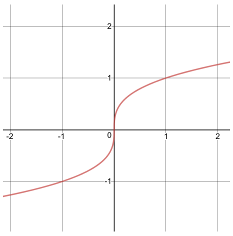
Power functions

Strictly increasing: $f(x_2)>f(x_1)$ where $x_2>x_1$ (including $x=0$)

Odd and even functions

Even when $f(x)=-f(x)$
Odd when $-f(x)=f(-x)$

Function is even if it can be reflected across y-axis $\implies f(x)=f(-x)$
Function $x^{\pm\frac{p}{q}}$ is odd if q is odd

	n is even	n is odd
x^n, $n \in \mathbb{Z}^+$		
x^n, $n \in \mathbb{Z}^-$		
$x^{\frac{1}{n}}$, $n \in \mathbb{Z}^+$		

$x^{\frac{-1}{n}}$ **where** $n \in \mathbb{Z}^+$

Mostly only on CAS.

We can write $x^{\frac{-1}{n}} = \frac{1}{x^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{x}}$n.

Domain is: $\begin{cases} \mathbb{R} \setminus \{0\} & \text{if } n \text{ is odd} \\ \mathbb{R}^+ & \text{if } n \text{ is even} \end{cases}$

If n is odd, it is an odd function.

$x^{\frac{p}{q}}$ **where** $p, q \in \mathbb{Z}^+$

$$x^{\frac{p}{q}} = \sqrt[q]{x^p}$$

- if $p > q$, the shape of x^p is dominant
- if $p < q$, the shape of $x^{\frac{1}{q}}$ is dominant
- points $(0, 0)$ and $(1, 1)$ will always lie on graph
- Domain is: $\begin{cases} \mathbb{R} & \text{if } q \text{ is odd} \\ \mathbb{R}^+ \cup \{0\} & \text{if } q \text{ is even} \end{cases}$

Piecewise functions

$$\text{e.g.} \quad f(x) = \begin{cases} x^{1/3}, & x \leq 0 \\ 2, & 0 < x < 2 \\ x, & x \geq 2 \end{cases}$$

Open circle: point included

Closed circle: point not included

Operations on functions

For $f \pm g$ and $f \times g$: $\quad \text{dom}' = \text{dom}(f) \cap \text{dom}(g)$

Addition of linear piecewise graphs: add y-values at key points

Product functions:

- product will equal 0 if $f = 0$ or $g = 0$
- $f'(x) = 0 \veebar g'(x) = 0 \not\Rightarrow (f \times g)'(x) = 0$

Composite functions

$(f \circ g)(x)$ is defined iff $\text{ran}(g) \subseteq \text{dom}(f)$