

# Dynamics

## 1 Resolution of forces

**Resultant force** is sum of force vectors

### 1.1 In angle-magnitude form

Cosine rule:  $c^2 = a^2 + b^2 - 2ab \cos \theta$       Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

### 1.2 In $i$ — $j$ form

Vector of  $a$  N at  $\theta$  to  $x$  axis is equal to  $a \cos \theta \mathbf{i} + a \sin \theta \mathbf{j}$ . Convert all force vectors then add.  
To find angle of an  $a\mathbf{i} + b\mathbf{j}$  vector, use  $\theta = \tan^{-1} \frac{b}{a}$

### 1.3 Resolving in a given direction

The resolved part of a force  $P$  at angle  $\theta$  is has magnitude  $P \cos \theta$

To convert force  $\|\vec{OA}$  to angle-magnitude form, find component  $\perp \vec{OA}$  then  $|\mathbf{r}| = \sqrt{(\|\vec{OA}\|)^2 + (\perp \vec{OA})^2}$ ,  $\theta = \tan^{-1} \frac{\perp \vec{OA}}{\|\vec{OA}\|}$

## 2 Newton's laws

1. Velocity is constant without a net external velocity
2.  $\frac{d}{dt} \rho \propto \Sigma F \implies \mathbf{F} = m\mathbf{a}$
3. Equal and opposite forces

### 2.1 Weight

A mass of  $m$  kg has force of  $mg$  acting on it

### 2.2 Momentum $\rho$

$$\rho = mv \quad (\text{units kg m/s or Ns})$$

### 2.3 Reaction force $R$

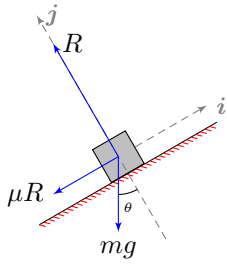
- With no vertical velocity,  $R = mg$
- With upwards acceleration,  $R - mg = ma$
- With force  $F$  at angle  $\theta$ , then  $R = mg - F \sin \theta$

### 2.4 Friction

$$F_R = \mu R \quad (\text{friction coefficient})$$

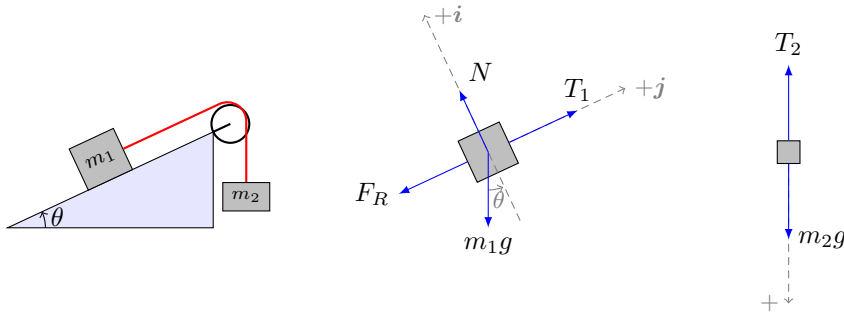
### 3 Inclined planes

$$\mathbf{F} = |\mathbf{F}| \cos \theta \mathbf{i} + |\mathbf{F}| \sin \theta \mathbf{j}$$



### 4 Connected particles

- **Suspended pulley:** tension in both sections of rope are equal
- **Linear connection:** find acceleration of system first
- **Pulley on edge of incline:** find downwards force  $W_2$  and components of mass on plane



### 5 Equilibrium

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \quad \text{(Lami's theorem)}$$

Three methods:

1. Lami's theorem (sine rule)
2. Triangle of forces or CAS (use to verify)
3. Resolution of forces ( $\Sigma F = 0$  - simultaneous)

On CAS: use Geometry, lock known constants.