MATHEMATICS

SUPPORT CENTRE

Title: Factorisation of quadratics.

Target: On completion of this worksheet you should be able to factorise any quadratic expression.

A **factor** is a term that divides an expression leaving no remainder.
<u>Examples.</u>
1. 3 is a factor of 6.
2. 1, x, 3, x^2, $3x$, $3x^2$ are factors of $3x^2$.

When we **factorise** an expression we write it as a product some of its factors.
E.g. $6 = 3 \times 2$.
There are three main methods for factorising quadratic expressions.

- By finding a **common factor**.
- By identifying the expression as the **difference of two squares**.
- By considering **guessing the factors** and checking by expanding the brackets.

To find a **common factor**, in an algebraic expression, we must first find the factors of its' constituent terms. Then we find the (largest) factor that its terms have in common.

E.g. Find the largest common factor of $3x+6x^2$.

$3x$ has factors 1, 3, x and $3x$,
$6x^2$ has factors 1, 2, 3, 6, x, $2x$, $3x$, $6x$, x^2, $2x^2$, $3x^2$, $6x^2$.
The largest factor they have in common is $3x$.

To <u>factorise an expression with a common factor</u> we need to find what the common factor should be multiplied by in order to get the original expression.

<u>Examples.</u>
1. Factorise $3x + 6x^2$.
$$3x + 6x^2 = 3x(1 + 2x).$$
2. Factorise $xy^2 - yz$.
$$xy^2 - yz = y(xy - z).$$

See the algebra sheet on common factors if you have difficulty with this.

<u>Exercise.</u>
Factorise the following expressions by finding a common factor. In each case check your answer by expanding the brackets.
1. $4x^2 + 2x$.
2. $3x^2 - 6x$.
3. $2x^2 - 4x$.
4. $6r^2 + 4r$.
5. $4sw^2 - sw$.
6. $4x^2 - 5x$.
7. $r(x + y)^2 + p(x + y)$.

(Answers: $2x(2x+1)$, $3x(x-2)$, $2x(x-2)$, $2r(3r+2)$, $sw(4w-1)$, $x(4x-5)$, $(x+y)(r(x+y)+p)$.)

Sometimes an algebraic expression does not have any common factors. We must then try one of the other methods.

A **perfect square** is a term that has a square root.
E.g. 9, x^2 and $4y^2$ are all perfect squares.

See the algebra sheet on indices if you have difficulty with this.

If an algebraic expression consists of two perfect squares one subtracted from the other then we say it is the **difference of two squares**. We can factorise these expressions simply by first finding the square root of its' terms.
<u>Example.</u>

One plus and one minus sign.

$$x^2 - y^2 = (x - y)(x + y).$$

Square root of 1st term. *Square root of 2nd term.*

<u>Example.</u>
$$4z^2 - 9 = (2z-3)(2z+3).$$

Exercise.
Factorise the following. Check your answers by expanding the brackets.

1. $y^2 - s^2$.
2. $z^2 - 16$.
3. $9 - p^2$.
4. $4r^2 - 25s^2$.
5. $x^2y^2 - 36r^2$
6. $(x + y)^2 - y^2$.

(Answers: $(y - s)(y + s)$, $(z - 4)(z + 4)$, $(3 - p)(3 + p)$, $(2r - 5s)(2r + 5s)$, $(xy - 6r)(xy + 6r)$, $x(x + 2y)$.)

When a quadratic algebraic expression has no common factors and is not the difference of two squares we must **guess the factors**. We know that it must be written as the product of two brackets. We should then check our guess by expanding the brackets.
Clearly this could take a long time. To make a sensible guess we should consider the following.

Multiply a and b together to get ab.

$$(x+a)(x+b)=x^2+(a+b)x+ab.$$

Add a and b together to get the coefficient of x.

Example
Factorise 1. $x^2 + 9x + 20$ and 2. $y^2 - 2y - 8$.

1. The possible values of ab satisfying ab=20 are 1×20, 2×10, and 5×4.
   Of these only 5 and 4 add together to give 9, therefore
   $$x^2 + 9x + 20 = (x + 4)(x + 5).$$

2. The possible values of ab satisfying ab=-8 are 1×-8, 2×-4, 4×-2 and 8×-1.
   Of these only 2 and –4 add together to give –2 therefore
   $$y^2 - 2y - 8 = (y - 4)(y + 2).$$

Exercise. Factorise the following:

1. $y^2 + 7y + 12$.
2. $x^2 + 6x + 9$.
3. $r^2 + 15r + 36$.
4. $x^2 - 8x + 16$.
5. $y^2 - 4y - 32$.
6. $p^2 + p - 12$.
7. $z^2 + 30z - 64$

(Answers: $(y+3)(y+4)$, $(x+3)(x+3)$, $(r+12)(r+3)$, $(x-4)(x-4)$, $(y-8)(y+4)$, $(p+4)(p-3)$, $(z + 32)(z-2)$.)

When the coefficient of x^2 is not one we have to guess the factors more carefully.
We should consider the following.

Multiply p and q together to get the coefficient of x^2.

$$(px+a)(qx+b)=pqx^2+(pb+qa)x+ab.$$

Multiply a and b together to get ab.

The following process is helpful.

- List the possibilities for $p \times q$.
- List the possibilities for $a \times b$.
- Try each possible pair of brackets and check them by expanding the brackets.

Example. Factorise $2x^2 + 11x + 12$.

- Possible values of $p \times q$ are 2×1.
- Possible values of $a \times b$ are 1×12, 2×6, 3×4, 4×3, 6×2, 12×1. (Notice that the ordering matters)
- Try $(2x+1)(1x+12)$. Expanding gives $2x^2+25x+12$, so this is wrong.
  Try $(2x+2)(1x+6)$. Expanding gives $2x^2+14x+12$, so this is wrong.
  Try $(2x+3)(1x+4)$. Expanding gives $2x^2+11x+12$ so this is correct.

Therefore,
$$2x^2 + 11x + 12 = (2x + 3)(x + 4).$$

Example. Factorise $3x^2 + 25x - 18$.

- 3×1.
- 1×-18, 2×-9, 3×-6, 6×-3, 9×-2, 18×-1, -18×1, -9×2, -6×3, -3×6, -2×9, -1×18.
- Trying the possibilities $(3x+1)(1x-18)$, $(3x+2)(x-9)$, etc gives us

$$3x^2 + 25x - 18 = (3x - 2)(1x + 9).$$

Exercise. Factorise the following:

1. $3x^2 + 11x + 6$.
2. $5x^2 + 36x + 7$.
3. $7x^2 + 26x - 8$.
4. $3x^2 - 13x + 12$.
5. $2x^2 + 2x - 12$.

(Answers: $(3x+2)(x+3)$, $(5x+1)(x+7)$, $(7x-2)(x+4)$,$(3x-4)(x-3)$,$(2x+6)(x-2)$.)