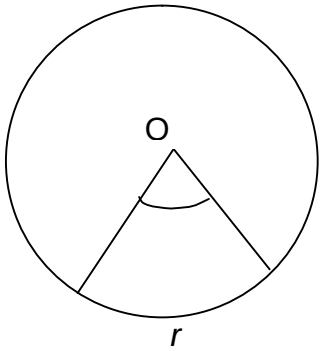
T4

MATHEMATICS SUPPORT CENTRE

Title: Radian Measure

Target: On completion of this worksheet you should understand what a radian is, be able to convert between degrees and radians and perform calculations using radians.

In the same way as a length can be measured in inches or in centimetres there is more than one way to measure an angle. So far we have used degrees but **radians** are another measure.
Consider a circle of radius *r* with an arc of length *r* and centre at O.



The angle marked is defined to be **one radian**. We say that this is the angle subtended at the centre by the arc. All circles are similar so the angle is the same whatever the radius of the circle.

Example
A circle has a radius of 8cm. An arc of length 16cm is drawn. What is the angle subtended at the centre by this arc?

Each arc of length 8cm gives an angle at the centre of 1 radian. So as 16 ÷ 8 = 2 the angle subtended at the centre is 2 radians.

Exercise
What is the angle subtended at the centre of a circle of radius 3cm by an arc of length 9cm? (Answer: 3 radians)

Exercise
Convert the following into radians (leave your answers in terms of π)
1. 90^0 2. 30^0 3. 270^0

We need to be able to convert between degrees and radians so we will find out how many radians there are in a circle of radius *r*.

The number of radians in a circle is equal to the number of arcs of length *r*. The circumference of the circle is $2\pi r$ so number of radians = $2\pi r / r = 2\pi$
But there are 360^0 in a circle so

2π radians = 360^0
π radians = 180^0
1 radian ≈ 57^0

Examples
1. Convert 60^0 into radians.

$$180^0 = p \text{ radians}$$
$$1^0 = \frac{p}{180} \text{ radians}$$
$$60^0 = \frac{p}{180} \times 60 \text{ radians}$$
$$60^0 = \frac{p}{3} \text{ radians}$$

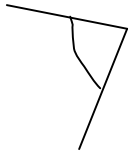
In this case we have left the angle in terms of π.

2. Convert 32^0 into radians

$$180^0 = p \text{ radians}$$
$$1^0 = \frac{p}{180} \text{ radians}$$
$$32^0 = \frac{p}{180} \times 32 \text{ radians}$$
$$32^0 = 0 \cdot 559 \text{ radians}$$

We have calculated the answer to 3 dp.

Suppose we have a circle of radius *r* cm with an arc subtending an angle of 1 radian at the



Examples

1. Convert $\frac{p}{5}$ radians into degrees.

p radians $= 180^0$

$1 \text{ radian} = \frac{180^0}{p}$

$\frac{p}{5}$ radians $= \frac{180}{p} \times \frac{p}{5}$ (note that p cancels)

$= 36^0$

2. Convert 1·3 radians into degrees.

p radians $= 180^0$

$1 \text{ radian} = \frac{180^0}{p}$

$1 \cdot 3 \text{ radians} = \frac{180}{p} \times 1 \cdot 3$

$= 74 \cdot 5^0$

Exercise

Convert the following into degrees:

1. $\frac{p}{2}$ radians
2. $\frac{p}{3}$ radians
3. $\frac{p}{4}$ radians
4. $\frac{3p}{2}$ radians
5. $\frac{5p}{6}$ radians
6. $\frac{7p}{3}$ radians
7. 0·78 radians
8. 1·2 radians
9. 4·9 radians
10. 2·56 radians
11. 3·14 radians
12. 7·02 radians

(Answers: 90^0, 60^0, 45^0, 270^0, 150^0, 420^0, $44 \cdot 7^0$, $68 \cdot 8^0$, $280 \cdot 7^0$, $146 \cdot 7^0$, $179 \cdot 9^0$, $402 \cdot 2^0$)

Examples

A circle has a radius of 65mm and an arc subtends an angle of 1·6 radians at the centre. Find the length of the arc and the area of the sector.

Length of arc = $65 \times 1 \cdot 6 = 104$mm

Area of sector = $\frac{1}{2} \times 65^2 \times 1 \cdot 6 = 3380\text{mm}^2$

Exercises

Find the length of the arc and the area of the sector for the following circles:

No.	Radius	Angle at Centre
1	7cm	2 radians
2	2m	0·9 radians
3	78mm	$\pi/3$ radians

(Answers: 14cm, 49cm^2; 1·8m, $1 \cdot 8\text{m}^2$; 81·7mm, 3190mm^2)

We often leave out the word 'radians' when it is clear that the angle is measured in radians. For example we write cos π and assume we are working in radians. To evaluate this, change your calculator into **radian mode** and then either enter ***π* cos** or **cos *π* =** in either case the answer is **-1**. The method used depends on your type of calculator. Similarly to find $\sin\left(\frac{p}{3}\right)$ either $p \div 3 = \sin$ or $\sin(p \div 3) =$ to give the answer 0·866