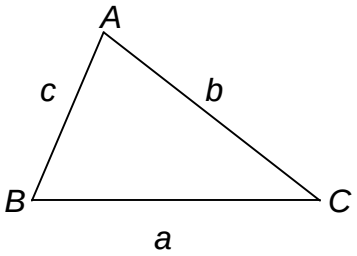
T7

MATHEMATICS SUPPORT CENTRE

Title: Cosine Rule

Target: On completion of this worksheet you should be able to use the cosine rule to find the sides and angles of a triangle.

The cosine rule can be used to find the sides and angles of a triangle when the sine rule cannot be used.



The cosine rule states:

$a^2 = b^2 + c^2 - 2bc\cos A$

or $b^2 = a^2 + c^2 - 2ac\cos B$

or $c^2 = a^2 + b^2 - 2ab\cos C$

These can be rearranged to give

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Note that there is no ambiguous case (see sine rule) when we use the cosine rule as any angle greater than 90^0 will have a negative cosine.

Examples

The following examples refer to the triangle above.

1. $A = 60^0$, b = 8cm and c = 5cm. Find a.

Using $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 60^0$

$a^2 = 49$

$a = \sqrt{49}$

a =7cm

Examples cont

2. a = 31mm, b = 45mm and c = 59mm. Find the angles of the triangle.

We will find the largest angle first in case it is obtuse (greater than 90^0). The largest angle is opposite the shortest side.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{31^2 + 45^2 - 59^2}{2 \times 31 \times 45}$$

$\cos C = -0\cdot1774$ (so $C > 90^0$)

$C = \cos^{-1}(-0\cdot1774)$

$C = 100\cdot2^0$

We will use the sine rule to find one of the other angles, say A

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{31} = \frac{\sin 100\cdot2^0}{59}$$

$$\sin A = \frac{\sin 100\cdot2^0}{59} \times 31$$

$\sin A = 0\cdot5171$

$A = \sin^{-1} 0\cdot5171$

$A = 31\cdot1^0$

$B = (180 - 100\cdot2 - 31\cdot1)^0$

$B = 48\cdot7^0$

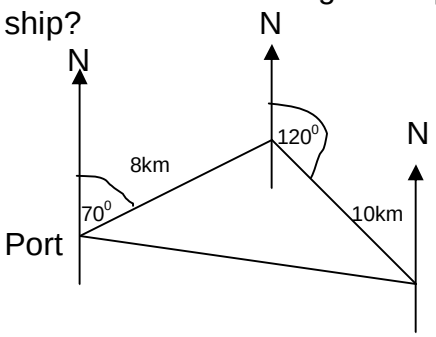
Note: The cosine rule together with the sine rule (see sheet T6) will solve any triangle if you are given any three values from either the sides or the angles of the triangle.

Exercise

Solve the following triangles. All questions refer to the triangle overleaf (lengths in mm)

No.	*A*	*B*	*C*	*a*	*b*	*c*
1			80^0	15	17	
2	63^0				92	85
3		32^0		23		46
4				73	80	89
5				112	203	160

6. A ship sails from port on a bearing of 070^0 a distance of 8km. It then changes course to a bearing of 120^0 and sails a further 10 km. How far is it from the port and what is the bearing of the port from the ship?



(Answers:

No.	*A*	*B*	*C*	*a*	*b*	*c*
1	**45.7^0**	**54.3^0**	80^0	15	17	**20.6**
2	63^0	**62.2^0**	**54.8^0**	**92.7**	92	85
3	**24.7^0**	32^0	**123.3^0**	23	**29.2**	46
4	**50.8^0**	**58.2^0**	**71.0^0**	73	80	89
5	**33.4^0**	**94.9^0**	**51.7^0**	112	203	160

6. 16.3km, 278^0)